1. (1) For equation (a): Is it linear or non-linear? Is it homogeneous or non-homogeneous? What is its order? (5 %)

$$z_{tt} + (\cos z)_t = 0 \qquad \text{(a)}$$

(2) Solve the differential equation (b). (10 %)

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0 \qquad \text{(b)}$$

2. Consider the first order differential equation

$$x^2(\partial_x \varphi)(x,y) - y^2(\partial_y \varphi)(x,y) = 0 \qquad \text{(c)}$$

with boundary condition 'at infinity' $\varphi(x,y) \to e^{1/x}$ as $y \to \infty$

(1) The equation (c) can be written

$$G(x,y) \cdot \nabla\varphi(x,y) = 0$$

where $G(x,y) = (x^2, -y^2)$. Sketch the vector-field G. (This diagram does not need to be too accurate, just as long as it contains the relevant information.) (5 %)

(2) Solve equation (c) using a change of variables. (5 %)

(3) Explain where your change of variables breaks down and how this is consistent with your diagram of G. (5%)

(4) Use variables separation (that is, write $\varphi(x,y) = X(x)Y(y)$ in (c)) to find a solution to (c) with the prescribed boundary conditions. (10 %)

3. Given,

$$\mathbf{B} = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Find all matrices associated with the diagonal decomposition, $\mathbf{B} = \mathbf{PDP}^{-1}$, of **B**, where **D** is the diagonal matrix formed from the eigenvalues of **B**, and the columns of **P** are the corresponding eigenvectors of **B**. (10%)

4. (1) What is the Fourier transform? (5 %)

(2) What is the Laplace transform? (5 %)

(3) Please show that: The Laplace transform is a natural result of providing the Fourier transform with a built-in convergence factor. (10 %)

5. Determine those values λ for which the following set of equations may possess a nontrivial solution.

$$3X_1 + X_2 - \lambda X_3 = 0$$

$$4X_1 - 2X_2 - 3X_3 = 0$$

$$2\lambda X_1 + 4X_2 + \lambda X_3 = 0$$

For each permissible value of λ, determine the most general solution. (15 %)

6. The symmetric matrix $\mathbf{A} = [a_{ij}]$ is a square matrix for which $a_{ji} = a_{ij}$. Let B and **C** represent symmetric matrices of order n. Prove that **BC** is also symmetric if and only if B and **C** are commutative. (15 %)

試題隨卷繳回