

1. Find the $PA = LDU$ factorization for $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$. (20%)
2. Write all known relations between the rank r and the dimension of $A_{m \times n}$ if the $A\bar{x} = \bar{b}$ has
 - (a) no solution for some b .
 - (b) infinitely many solutions for every b .
 - (c) exactly one solution for some b , no solution for other b .
 - (d) exactly one solution for every b . (20%)
3. Let $A = [3 \ 1 \ -1]$, and let V be the nullspace of A .
 - (a) Find an orthonormal basis for V and an orthonormal basis for V^\perp (perpendicular to V).
 - (b) Find the projection matrix that projects vectors in \mathbb{R}^3 onto V^\perp . (20%)
4. Find a best approximation to $y = x^4$ by a straight line between $x = 0$ and $x = 1$. (20%)
5. Find the general solution to $du/dt = Au$ if

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Can you find a time T at which the solution $u(T)$ is guaranteed to return the initial value $u(0)$? (20%)