(1) (10%) Consider a family of curves that are graphs of $F(x,y)=y-kx^3=0$. Please find the family of orthogonal trajectories.

(2) (10%) Verify that the given function is a solution of the differential equation, find a second solution by reduction of order, and then write the general solution.

$(2x^2+1)y''-4xy'+4y=0,\quad y_1(x)=x$ for $x>0$

(3) (20%)The Laplace transform of f is defined as $F(z)=\int_0^\infty e^{-zt} f(t)\,dt$ for all z such that the integral is defined and finite. Now if F be differentiable for all z except for a finite number of points $z_1,\cdots,z_n$, which are all poles of F. Then the inverse Laplace transform of $F(z)$ can be expressed as $f(t)=\sum_{j=1}^{n}\mathrm{Res}[e^{zt}F(z), z_j]$. Use the formula to find the inverse Laplace transform of function $F(z)=\dfrac{1}{(z+1)(z-3)^2}$. (hint: Res means residues).

(4) (20%) Prove that the eigenvalues of a Hermitian matrix are real.

(5) (20%) Determine the location and type of all critical points by linearization.

$$\begin{cases} x'=y \\ y'=-\dfrac{k}{m}x+\dfrac{\alpha}{m}x^3-\dfrac{c}{m}y \end{cases}$$

(6) (20%) If the general solution of the equation $x^2y''+xy'+(x^2-v^2)y=0$ can be expressed by $y(x)=A\,J_v(x)+B\,J_{-v}(x)$

Fine the General solution (in terms of the Bessel function) of the equation:

$$x^2y''+(1-2v)xy'+v^2(x^{2v}+1-v^2)y=0$$

Hint: $u=x^{-v}y \qquad z=x^v$

試題隨卷繳回