*Note: 請將題號及答案標示清楚

1. (25%) Consider a system: $G:\begin{cases}\dot{x}=\begin{bmatrix}-3 & 1 & 0\\ 0 & -3 & 0\\ 0 & 1 & 1\end{bmatrix}x+\begin{bmatrix}0\\1\\1\end{bmatrix}u\\ y=\begin{bmatrix}1 & 1 & 1\end{bmatrix}x\end{cases}$

(1) (5%) Please find the state response $x(t)$ to $x(0)=\begin{bmatrix}1 & 1 & 0\end{bmatrix}^T$ and $u(t)=0$.

(2) (5%) Please calculate the transfer function of $G(s)$.

(3) (5%) Consider the closed-loop system of Figure 1, with a proportional controller $C(s)=K$. Please sketch the root-loci of the closed-loop poles as $K=0\to\infty$.

(4) (5%) Find the minimum value of $K=K_0$ such that the closed-loop system is stable.

(5) (5%) Let $K=4K_0$, find the steady-state error of the closed-loop system to a unit step input R.

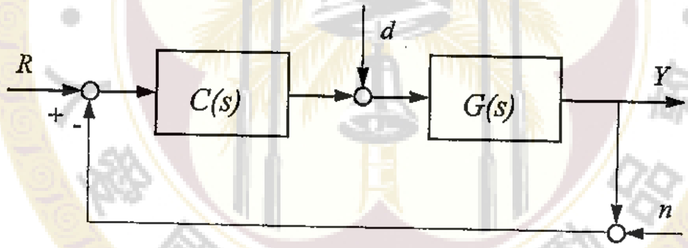


Figure 1: A closed-loop system.

2. (25%) Refer to Figure 1 with $G(s)=\dfrac{s}{s^2-1}$.

(1) (5%) Can the system be stabilized by *P*-control $C(s)=K$? Explain your answer by root-locus.

(2) (5%) Apply a first-order controller $C(s)=\dfrac{K(s+b)}{s-a}$, please find the conditions (in terms of K, a, b) to achieve closed-loop stability.

(3) (5%) Continued from (2), find the minimum achievable settling time (in terms of a, b) to a unit step input R.

(4) (10%) Set $C(s)=\dfrac{K(s+1)}{s-1}$, find the minimum value of K such that the system's damping ratio is greater than $1/2$.

3. (25%) A NTU teaching assistant, Jay Chou, was preparing a paper illustrating the correlation among Nyquist plots and frequency responses of closed-loop systems. Unfortunately, he dropped the material on the floor and got the graphs all mixed up. The graphs are shown on the next page. *You must help Jay identify which Nyquist plot corresponds to which frequency response plot.* The block diagram of the feedback system is shown below.

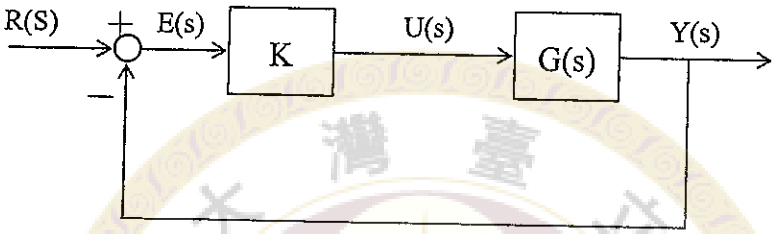
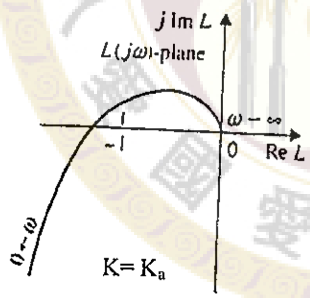


Figure 2: Diagram of a closed-loop system

In Figure 2, G(s) is the transfer function of a typical third-order, minimum-phase system, and K is the loop gain. Let *L(s)* be the loop transfer function of the above closed-loop system, and M(jw) represent the frequency response of the closed-loop system. The Nyquist plots for L(s) with different positive values of K (K_a, K_b, K_c, K_d) are shown in Figures (a)~(d):



Figure(a)

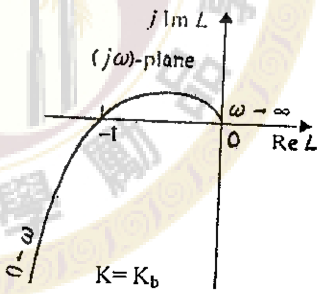
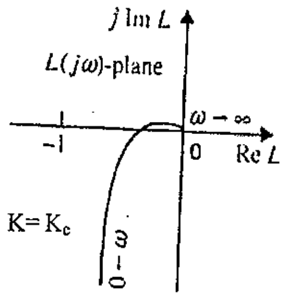


Figure (b)



Figure(c)

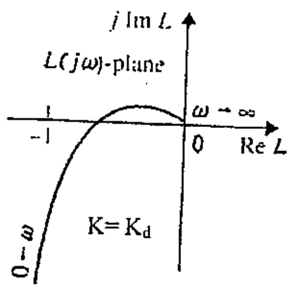


Figure (d)

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The frequency responses of the closed-loop system with different positive values of K (K_a, K_b, K_c, K_d) are shown in Figures (e)~(h):

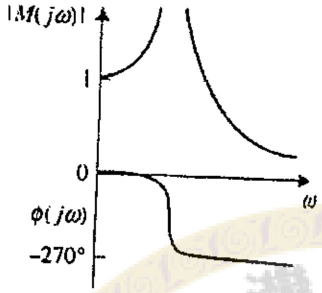

Figure (e)

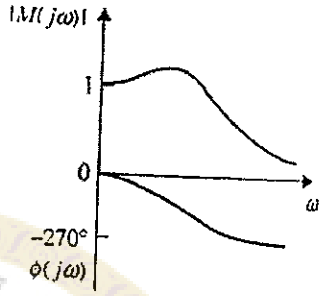

Figure (f)

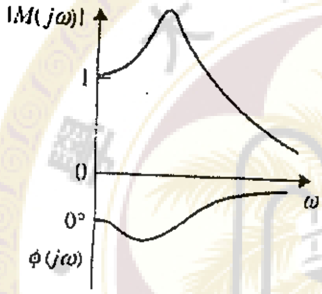

Figure (g)

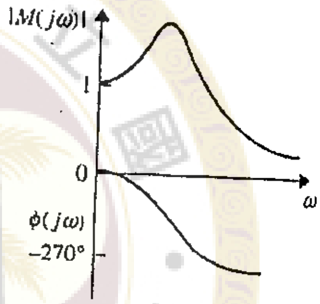

Figure (h)

(1) (5%) From Figures (a)~(h), you are able to assess the **relative stability** of a system in the frequency domain. Please determine which Nyquist plot, Figure (a)~(d), corresponds to which closed-loop frequency response plots, Figure (e)~(h).

(2) (5%) Figure (i) below shows the step response for the above closed-loop system with a loop gain K_i, which is the same as one of the K values in Figure (a)~(d). Please determine the value of K_i in terms of K_a, K_b, K_c, and K_d. **Also, determine the stability of the closed-loop system with this loop gain K_i.**

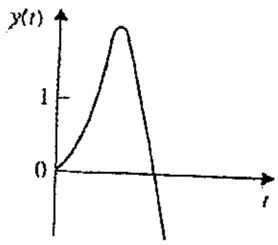

Figure (i)

(3) (5%) In Figures (e)~(h), is there any system a marginally unstable system? If your answer is "yes", please determine which figure among Figures (e)~(h) shows the frequency response of this system. **Also, sketch the step response plot for this marginally unstable system.**

(4) (5%) By comparing Figures (a)~(d) and Figures (e)~(h), the relationship between relative stability and the **closed-loop** frequency response of the system with different values of loop gain can be found. Please make your comments about this relationship. (Note: You should mention the correlation of relative stability with both magnitude and phase response curves in Figures (e)~(h).)

(5) (5%) Suppose that G(s) in Figure 2 is a second-order, minimum-phase system, will the change of the value of loop gain K affect the relative stability of the system? Explain your answers!

4. (15%) A mass-spring-damper (m-k-c) system is described by the mathematical model:

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

where *f(t)* is the externally applied force. Let the damping ratio of this system be denoted by ζ. Figure 4 shows the step response of the system in terms of the displacement of the mass, x(t).

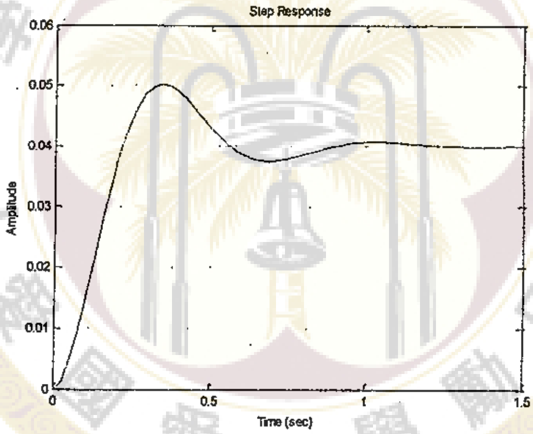

Figure 4: Time response of the displacement of mass, x(t), to a unit-step input

(1) (5%) Let M denote the amplitude ratio of the displacement response, $X(\omega)$, to the applied force, $F(\omega)$, in the frequency domain, i.e., $M = \frac{|X(\omega)|}{|F(\omega)|}$, where ω represents frequency.

Show that $M = \frac{1}{k}\frac{1}{2\zeta\sqrt{(1-r^2)^2+(2\zeta\cdot r)^2}}$, where $r = \omega\sqrt{m/k}$.

(2) (5%) Show that the peak value of M, denoted M_r, occurs when $0 \le \xi \le 0.707$ and $r = \sqrt{1-2\xi^2}$, and is $M_r = \frac{1}{k}\frac{1}{2\xi\sqrt{1-\xi^2}}$.

(3) (5%) Figure 5 below shows the Bode plot for this mass-spring-damper system. Suppose

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that the applied force, *f(t)*, is represented by a Fourier series as follows:

$$f(t) = -(\sin 3t + \frac{1}{3}\sin 9t + \frac{1}{5}\sin 15t + \frac{1}{7}\sin 21t + \cdots \frac{1}{n}\sin 3nt \pm \cdots), n \text{ odd.}$$

Determine the **bandwidth** of the system and find an approximate description of the output $x_{ss}(t)$ at steady state using only those input components that lie within the bandwidth.

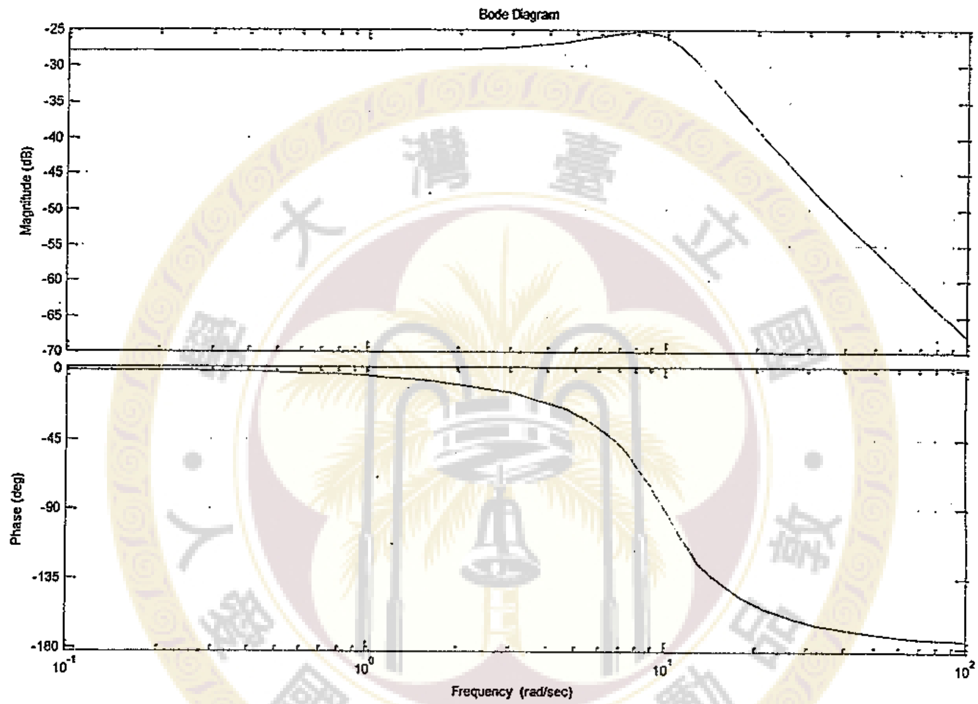

Figure 5: Bode plot of the mass-spring-damper system

5. (10%) Consider that a feedback control system has a loop transfer function L(s). The Bode plot of L(s) is shown in Figure 6.

(1) (5%) Determine the gain margin, phase margin and stability of the system.

(2) (5%) Can you use a single *phase-lead* compensator to improve the phase margin and stability of the system? If not, how would you design a controller for this system?

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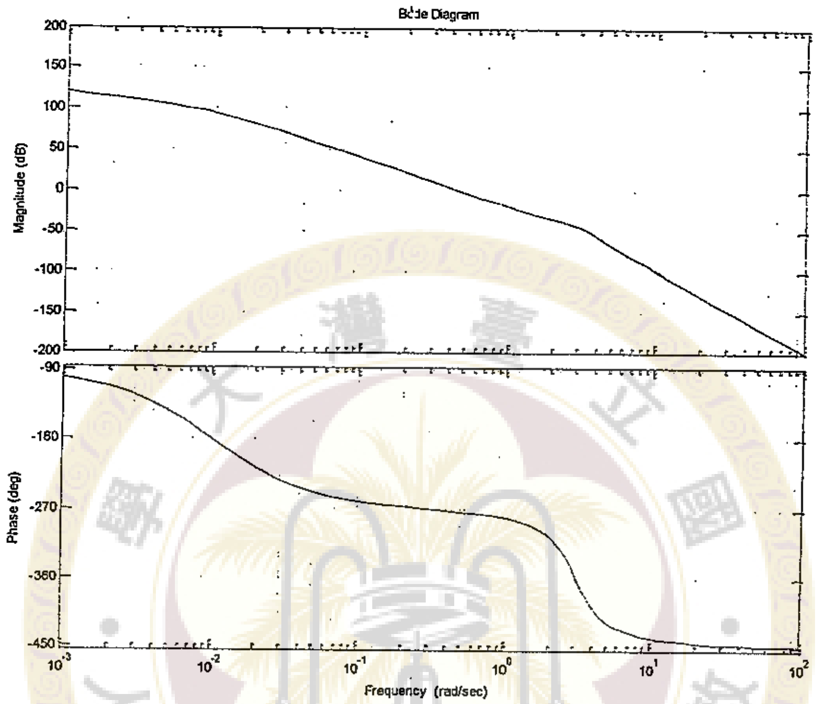

Figure 6: Bode plot of L(s)

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