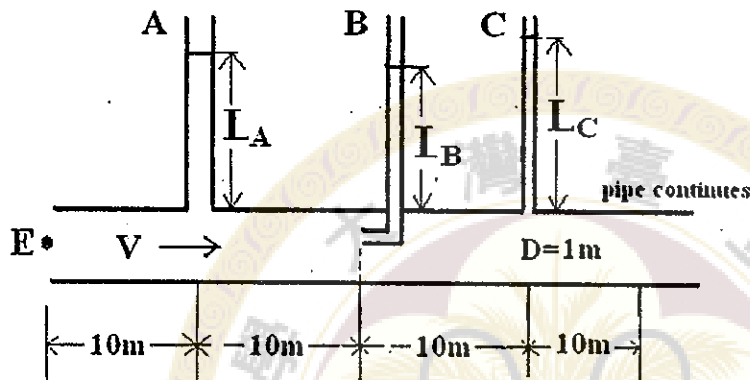


**Problem 1.** (36%) For the following figure, give the height of the water in pipe A, B and C at the steady state. The diameters of the pipes are 0.4m, 0.2m and 0.1m for pipe A, B, and C, respectively. The center line of the horizontal main pipe is considered as datum  $z=0$ . The pressure head at entrance (point E) is 8m at  $z=0$  regardless of velocity.

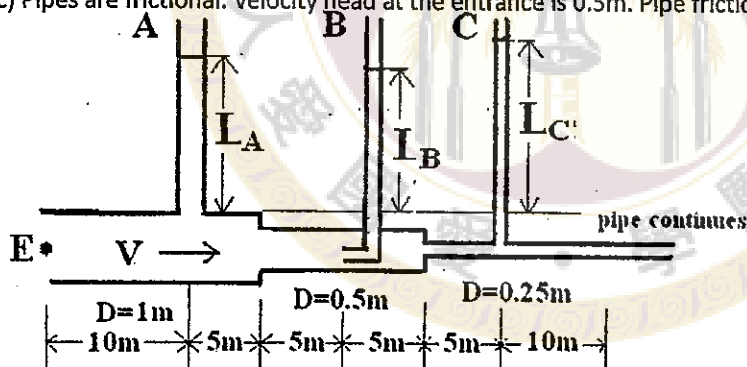
When you give your answers, you must write your calculations or explanations. Otherwise, there will be no point given. Also write your final answers as  $(L_A, L_B, L_C) = ( \quad , \quad , \quad )$  m.

For frictional pipes, consider friction loss only; neglect all other kinds of energy loss.



For the above figure, diameter for the main pipe is 1m. What is the height of water in pipe A, B and C for

- (a) Pipes are frictionless. Velocity head at the entrance is 0m
- (b) Pipes are frictionless. Velocity head at the entrance is 0.5m
- (c) Pipes are frictional. Velocity head at the entrance is 0.5m. Pipe friction factor=0.02



For the above figure, what is the height of water in pipes A, B and C for

- (d) Pipes are frictionless. Velocity head at the entrance is 0m
- (e) Pipes are frictionless. Velocity head at the entrance is 0.5m.
- (f) Pipes are frictional. Velocity head at the entrance is 0.5m. Pipe friction factor=0.02.

**Problem 2.** (25%) Given the velocity potential,  $\phi = U \cos \theta (r + \frac{a^2}{r})$  find

- (a) (6%) the stream function  $\psi$ .
- (b) (5%) Draw streamline  $\psi=0$ . Explain why streamlines can intersect.
- (c) For area  $r \geq a$ , draw the flow field with several stream lines (at least 3 lines and indicate the value of  $\psi$ ). (4%)
- (d) (3%) Follow (c), calculate the pressure at  $(0,a)$  and  $(-a,0)$  if the pressure at far away is P
- (e) for area  $r \leq a$ , draw the flow field with several stream lines (at least 3 lines and indicate the value of  $\psi$ ) (4%)
- (f) (3%) Follow (e), calculate the pressure at  $(0,a)$  and  $(-a,0)$  if the pressure at  $(0,0)$  is P

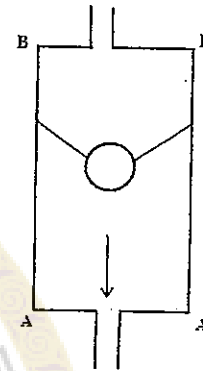
**Problem 3.** (14%) A spherical ball is tied to the wall within a big circular pipe with 2 strings. The radius of the ball is 0.3m and the specific gravity of the ball is 0.5. When the flow velocity  $V$  is 2m/s downward, the stresses on the strings are zero.

(a) (3%) The drag force is  $F_D = \frac{C_D}{2} \rho V^2 A$ , what is the value of drag coefficient  $C_D$

if the flow velocity is increased to 4m/s and we cut both strings at the same time.

(b) (3%) What will happen to the ball? ( $C_D$  is a constant)

(c) (8%) The ball eventually will hit the connection with the smaller pipes (either the lower one or the upper one). The ball can perfectly block the inflow or outflow. What is the force exerted on the surface at the moment when ball just hits the connection (The velocity in large pipe is the same, but in small pipe is zero). The diameter of the big pipe is 15m and the diameter of the small pipes (upper or lower) is 0.25m.



Hint: When you solve this problem, state clearly your conditions of calculation. There are several possible cases.

**Problem 4.** (25%) There is a Newtonian fluid with viscosity  $\mu$  flowing in an inclined channel. The bottom plate is stationary and the top plate is pulled at a constant speed  $U$  upward. The distance between two plates is a constant  $H$ . There is a constant pressure gradient  $P_0$  applied in  $x$  direction. For a **two-dimensional steady uniform flow** as shown in the figure. (Hint: You can solve (d) without solving (a), (b), (c))

(a) (6%) Prove that under steady uniform flow, the velocity in  $y$  direction is zero and the pressure distribution in  $y$  direction is static.

(b) (5%) Solve the velocity profile.

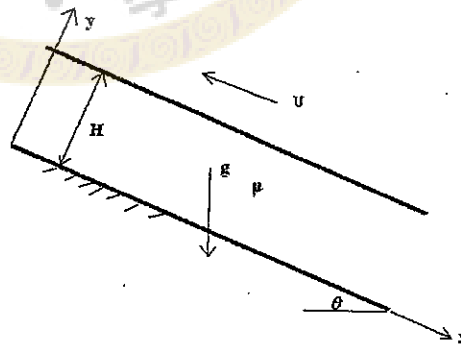
(c) (6%) If the net flow rate through this channel is zero, what is the relationship between  $U$ ,  $g$  and  $P_0$ .

(d) (8%) Follow (c), assuming you do not know the relation between  $U$  and  $P_0$ , try to use dimensional analysis to find the relationship between  $U$  and  $P_0$ .

Hint: The momentum equations are

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \rho g \sin \theta + \mu \nabla^2 u$$

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \rho g \cos \theta + \mu \nabla^2 v$$



試題隨卷繳回