In Problems 1-4 the bars are made of a homogeneous and isotropic, linearly elastic material with Young's modulus $E = 2.6$ GPa and the shear modulus $G = 1$ GPa.

1.(20%) A horizontal bar (shaft) has a length $L = 20$ m and circular cross-sections with outer radius 1 m — solid section ($0 \le r \le 1$ m) only on the left half span $0 \le x \le 10$ m while hollow section ($0.5 \le r \le 1$ m) on the right half span $10 \le x \le 20$ m. The bar is fixed at both ends and is subjected to a distributed torsional moment load $t = 31$ kN-m/m on the left half span $0 \le x \le 10$ m but no load on the right half span $10 \le x \le 20$ m.
(a) Find the reaction torques at the two ends.
(b) Calculate the twist angle at the midspan point $x = 10$ m.
(c) Determine the shear *strain* distribution over the circular hollow cross-section.

2.(20%) A vertical cantilever bar (column) has length $L = 20$ m and circular cross-section with radius 1 m. It is fixed at the lower end $x = 0$ and is subjected to a *compressive* load P at the upper end $x = 20$ m.
(a) Plot free-body diagrams and formulate the buckling equation and boundary conditions at the two ends (with detailed derivations).
(b) Determine the buckling load (with detailed solution process).
(c) Solve for the buckled mode shape (with detailed solution process) and plot it.

3.(20%) A horizontal cantilever bar (rod/beam) has a length $L = 2$ m and a thin-walled equal-leg angle section (等肢角鋼) with each leg 20 cm long and 1.5 cm thick. The angle section is placed such that one leg is horizontal and the other leg is vertical upwards. The bar is fixed at the left end $x = 0$ and is subjected to a *tensile* load 1 kN at the right end $x = 20$ m, but note that the tensile load acts horizontally and eccentrically on the uppermost tip of the vertical leg.
(a) Locate the shear center and the centroid of the angle section.
(b) Draw the axial force diagram and the bending moment diagram(s).
(c) Determine the normal *stress* distribution on the angle section.

4.(20%) Two thin-walled circular hollow bars (tubes), one with a slit and one without, have the same (midline) radius r, thickness $t \ll r$ and length L. They are fixed at the left end $x = 0$ and are each subjected to a same amount of *torsional* load T at the right end $x = L$.
(a) Compare their twist angles at $x = L$.
(b) Compare their shear distributions over the cross sections.
(c) Determine the ratio of their maximum shear stresses.

5.(20%) Let E be Young's modulus, G the shear modulus, and ν Poisson's ratio. It is known that E and ν may be measured from a uniaxial stress test (*i.e.* the tension/compression test) and G from a pure shear test (or a torsional shear test of thin-walled circular tube).
(a) Give the definitions (how to measure) of E, ν and G.
(b) State the condition (what kind of materials) for which the formula $E = 2G(1 + \nu)$ is valid.
(c) Under that condition *prove* the formula by comparing the uniaxial stress test with the pure shear test and drawing Mohr's circles for stress and strain.