國立臺灣大學 102 學年度碩士班招生考試試題

1. (15 分) 假設台北市主計局為了解 25-30 歲年輕人就業的情形，主計局計畫每個月隨機抽取 5 個人調查其就業情形，若台北市 25-30 歲年輕人每個月的就業率為 P，每個月均不同，而為一均等分配(0,1)。
   (a).試求有 4 位就業，1 位失業的非條件機率(unconditional probability)為何?
   (b).試求 5 個人樣本的就業人數平均數。
   (c).試求 5 個人樣本的就業人數變異數。

2. (10 分) 設$X_1, X_2, \dots X_n$為抽取自平均數μ，變異數為σ^2的隨機樣本，令 $S^2 = \frac{\Sigma(X_i - \bar{X})^2}{n-1}, \hat{\sigma}^2 = \frac{1}{2}(X_1 - X_2)^2$。
   (a).試問S^2與$\hat{\sigma}^2$ 為σ^2的不偏估計式嗎? 請證明之。
   (b).若假設母體分配為一常態分配，請比較S^2與$\hat{\sigma}^2$的相對有效性。

3. (15 分) 某工廠有 A、B 兩種機器生產產品，現已知 A 機器每週的修理成本X為一常態分配平均數為μ_1變異數為σ^2，B 機器每週的修理成本Y為一常態分配平均數為μ_2變異數為$2\sigma^2$。現工廠有兩部 A 機器及一部 B 機器，為了瞭解工廠每週之平均修理成本。自修理 A 機器的成本中抽取$X_1, X_2, \dots X_n$之隨機樣本，自修理 B 機器的成本中抽取$Y_1, Y_2, \dots, Y_m$之隨機樣本。
   (a).請建立工廠每週機器的平均修理成本 95%的信賴區間(假設σ^2未知，n為小樣本)。
   (b).請建立σ^2在 $C.C.=1\text{-}\alpha$下的信賴區間。
   (c).假設$\sigma^2 = 10$, $m=n$。試求應至少抽取多少樣本數，使得信賴區間長度在 2 個單位之內?

4. (10 分)
   (a).假設$f(X;\theta)=(\theta+1)X^{\theta}, 0 < X < 1$，檢定假設$H_0: \theta = 2$, $H_1: \theta = 3$。設拒絕域為$X \le 0.1, X \ge 0.9$，試求該檢定的α與β。
   (b).根據過去經驗顯示考汽車駕照通過路考的機率為 0.5，某同學抽樣 50 位有駕照的路人，紀錄他們通過路考的次數，結果發現每人平均 2.5 次才通過路考，請在α =0.05 下，檢定考汽車駕照通過路考的機率是否小於 0.5。

見背面

5. (20分) The data analyze here consist of test scores and class sizes in 1999 in 420 California school districts that serve kindergarten through eighth grade. The test score, *TestScore*, is the districtwide average of reading and math scores for fifth graders. Class size is measured by constructing the following binary variable

$$D_i = \begin{cases} 1 & \text{if } STR_i \leq 20 \\ 0 & \text{if } STR_i > 20 \end{cases}$$

where STR_i stands for the student-teacher ratio in i-th district for $i = 1, 2, \ldots, 420$. Consequently, the observations are divided into two groups: small class size ($STR \leq 20$) and large class size ($STR > 20$). The following table contains the information about group means and standard deviations.

Class Size	Average score ($\overline{Y}$)	Std. dev. (s_Y)	N
Small ($STR \leq 20$)	657.4	19.4	238
Large ($STR > 20$)	650.0	17.9	182

The ordinary least squares (OLS) is used to estimate a line relating the student-teacher ratio to the test scores using the 420 observations, yielding the following result.

$$\begin{aligned} \widehat{TestScore} &= \quad \hat{\beta}_0 \quad + \quad \hat{\beta}_1 \quad \times D \\ & \quad (SE(\hat{\beta}_0)) \quad (SE(\hat{\beta}_1)) \\ &= 650.0 + \ ? \ \times D, \\ & \quad (1.3) \quad (?) \end{aligned}$$

where $\hat{\beta}_0$ and $\hat{\beta}_1$ are OLS estimates, and the heteroskedasticity-robust standard errors of the estimates are $SE(\hat{\beta}_0)$ and $SE(\hat{\beta}_1)$; that is, $\hat{\beta}_0 = 650.0$ and $SE(\hat{\beta}_0) = 1.3$.

(a) Calculate $SE(\hat{\beta}_1)$.

(b) Is the relationship between *TestScore* and D (binary variable for class size) statistically significant?

(c) Are the OLS estimators, $\hat{\beta}_0$ and $\hat{\beta}_1$, efficient among all estimators that are linear in *TestScore* and unbiased, conditional on STR? Explain.

Now, consider the following regression:

$$\widehat{TestScore} = \hat{\beta}_0 + \hat{\beta}_1 STR + \hat{\beta}_2 PctEL,$$

where $PctEL$ is the percentage of students in the district who are English learners. Econometricians have verified that the STR and PctEL are positively correlated and the asymptotic variance of $\hat{\beta}_1$ is

$$\sigma^2_{\hat{\beta}_1} = \frac{1}{420}\left[\frac{1}{1-\rho^2}\right]\frac{\sigma^2_{error}}{\sigma^2_{STR}},$$

where ρ is the population correlation between the STR and $PctEL$.

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(d) Comment on the following statements: "When STR and $PctEL$ are correlated, the variance of $\hat{\beta}_1$ is larger than it would be if STR and $PctEL$ were uncorrelated. Thus, if you are interested in β_1, it is best to leave $PctEL$ out of the regression if it is correlated with STR"

6. (10分) Consider the following data generating process

$$y_t = \alpha y_{t-1} + e_t$$
$$e_t = \rho e_{t-1} + v_t,$$

where y_t are observable, v_t are white noise, α, ρ are parameters, and we assume $\mathbb{E}[y_{t-1}v_t] = 0$, $\mathbb{E}[y_{t-1}e_{t-1}] = \mathbb{E}[y_t e_t]$, and $\mathbb{E}[e_t^2] = Var(e_t) = \sigma^2$. Find the condition under which the least-squares method can be used to consistently estimate α through the regression $y_t = \alpha y_{t-1} + e_t$.

7. (5分) Consider the regression model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + e_i,$$

where Y, X, e, and β are dependent variable, regressors, error term, and unknown parameters, respectively. A single restriction (null hypothesis) involving multiple coefficients can be tested using a transformation in which the original regression model is rewritten to turn the restriction under the null hypothesis into a restriction on a single regression coefficient. Now, use the trick mentioned above to transform the regression so that you can use a t-statistic to test the null hypothesis: $\beta_1 + c\beta_2 = 1$, where c is a known constant.

8. (15分) Consider the simple regression

$$y = \beta_0 + \beta_1 x + e. \tag{1}$$

We say x is endogenous if x is correlated with the error term e. Suppose that there is an instrumental variable z which is correlated with x but uncorrelated with e. Thus, we can write

$$x = \theta_0 + \theta_1 z + v$$

and $\theta_1 \neq 0$. Since θ_0 and θ_1 are unknown, econometricians use OLS to calculate $\hat{x} = \hat{\theta}_0 + \hat{\theta}_1 z$ together with the residual $\hat{v} = x - \hat{x}$. Now, replace the x in equation (1) with $\hat{x}$, yielding

$$\begin{aligned} y &= \beta_0 + \beta_1 \hat{x} + \beta_1 \hat{v} + e \\ &\equiv \beta_0 + \beta_1 \hat{x} + \gamma \hat{v} + e, \end{aligned} \tag{2}$$

where to reduce confusion, we let the coefficient of $\hat{v}$ be denoted as γ. If we omit $\hat{v}$ from equation (2), the regression becomes

$$y = \beta_0 + \beta_1 \hat{x} + e. \tag{3}$$

(a) If x is exogenous, are the OLS estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ in (2) and (3) unchanged? If x is exogenous, will the OLS estimator $\hat{\gamma}$ converge to β_1 in large samples? Explain all your answers.

(b) If x is endogenous, will $\hat{\gamma}$ converge to β_1 in large samples? Explain.

(c) Suggest a t-test testing for endogeneity of the regressor x according to the discussions in parts (a) and (b). (*Hint*: use the trick mentioned in the Question 7.)