1. Find the solution $(x_1(t), x_2(t), x_3(t), x_4(t))$ of the system [20 points]

$$\begin{cases} x_1' &= -x_2 \\ x_2' &= x_1 \\ x_3' &= -x_4 \\ x_4' &= 2x_1 + x_3, \end{cases}$$

with $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0, 1, 0, 0)$.

2. Let $(x_1(t), x_2(t), x_3(t))$ satisfy the system [20 points]

$$\begin{cases} x_1' &= -2x_2 + x_2x_3 - x_1^3 \\ x_2' &= x_1 - x_1x_3 - x_2^2 \\ x_3' &= x_1x_2 - x_3^3. \end{cases}$$

Construct one Liapunov function (some quadratic function) to show that the origin O is asymptotically stable.[15 points] Is the origin a sink?[5 points]

3. Let [20 points]

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

(a) Compute the matrices of the semisimple and nilpotent parts of A, i.e., $A = S + N$. What is the order of the nilpotent matrix N?[10 points]

(b) Compute

$$e^{At}, e^{Nt}.$$

[10 points]

4. Find the general solution of [20 points]

$$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 4y = x.$$

5. Suppose that $y(x)$ satisfies [20 points]

$$\frac{dy}{dx} \leq 3x^{-1}y - x \text{ for } x \geq 3,$$

and $y(3) \leq 9$. Prove that $y \leq x^2$ for $x \geq 3$.

試題隨卷繳回