※ 注意：請於試卷上依序作答，並應註明作答之大題及其題號。

Logic Part I

*本試題使用的邏輯符號如下(如果你使用其它邏輯符號系統，請註明出自哪一本教科書)：

(x) :全稱量限號(universal quantifier)

(∃x) :存在量限號(existential quantifier)

¬: 否定號(negation) ∧: 連言號(conjunction)

∨ : 選言號(disjunction) → :條件號(conditional)

↔ :雙條件句(biconditional)

一、請用述詞邏輯(predicate logic)的符號翻譯以下的語句(每題 3 分)

(1) 並非每一個用功的學生考試都會及格。
(Ax: x 是學生，Bx: x 用功，Cx: x 考試及格)
(2) 如果一個人不尊敬自己，那麼別人也不會尊敬他。
(Hx: x 是人，Rxy：x 尊敬 y)
(3) 沒有最大的質數
(Px: x 是質數，Gxy:x 大於 y)

二、請用自然演繹法(natural deduction)證明以下論證為有效(每題 5 分)

(4) P↔(Q∧R), ¬Q∨¬R, S→P ⊢ ¬S

(5) (x)(Cx→Ax),(∃x)(Bx∧¬Ax) ⊢ (∃x)(Bx∧¬Cx)

三、簡答題，任選兩題，每題 3 分

(6) 請舉例說明傳統邏輯中「矛盾」(contradictory)與「大反對」(contrary)之間的關係。
(7) 請舉例說明有效(valid)論證與健全(sound)論證之間的差別。
(8) 「人都有理性，所以有些人有理性」，你認為這是一個有效或是無效推論？請說明你如何判斷它有效或無效？

見背面

Logic　PART II（請任選 3-5 題、總分合計 25%以上作答。給分上限為 25 分。）

1. (10%) Let Γ be a set of formulae, and φ, a formula, of a formal language $\mathcal{L}_K$ suitable for the propositional calculus.

(a) What does each of the following sequents mean?
- (i) $\vDash \varphi$
- (ii) $\varphi \vDash$
- (iii) $\Gamma \vDash \varphi$

(b) Given a logical system, or a theory, say **T**, constructed out of the language $\mathcal{L}_K$, Explain what the following sequents mean respectively
- (i) $\vdash_T \varphi$
- (ii) $\varphi \vdash_T$
- (iii) $\Gamma \vdash_T \varphi$

(c) Sequents in (a) are known as semantic sequents, why they are so-called? By contrast, sequents in (b) are known as syntactic sequetns, why they are so-called? Some logicians, such as Hodges, claims that a semantic sequent and its corresponding syntactic sequent amount to the same things, although they were defined quite differently'. Would you agree with such a claim?

2. (10%) Let Γ be a set of formulae, φ and ψ, formulae, of a formal language $\mathcal{L}_K$ suitable for the propositional calculus.

(a) Show that $\Gamma \vDash (\varphi \to \psi)$ iff $\Gamma, \varphi \vDash \psi$.
(b) Consider the statement that $\Gamma \vDash (\varphi \vee \psi)$ iff either $\Gamma \vDash \varphi$ or $\Gamma \vDash \psi$. Is this true? If not, provide a counter-example.
(c) Consider the statement that $\Gamma \vDash \neg\varphi$ if it is not true that $\Gamma \vDash \varphi$. Is this true?

3. (10%) In ordinary language, there are singular terms which may have no reference in the actual world. Frege claimed that a sentence with any singular term of this sort would be neither true nor false. But Russell claims that a sentence of this sort must be false. Which one is more acceptable? Why? If both are not acceptable, any better treatment?

4. (5%) What is a substitutional instance of a given formula in the language $\mathcal{L}_K$? Now, assume that

$$(P \to (Q \to R)) \vdash (Q \to (P \to R))$$

Without constructing a derivation (or proof) in any specified formal system, show that the following sequent holds as well.

$$((Q \wedge R) \to (((P \vee Q) \to R) \to (P \to Q))) \vdash (((P \vee Q) \to R) \to ((Q \wedge R) \to (P \to Q))).$$

5. (5%) What is the principle of bivalence? The principle is said to be essential to the classical propositional calculus. Do you think that the rejection of this principle would falsify some theorems of the classical propositional calculus? If not, give an informal argument. If yes, provide an example (i.e. a theorem of the classical propositional calculus) and then show that it would fail to hold, if the principle is rejected.

接次頁

6. (5%) Predicate logic is sometime also called first-order logic. Why? And explain what a second-order logic is. Some logicians reject second-order logic. Do they have any good reasons?

7. (5%) In a novel the author usually introduces some main characters by so-called fictional names, such as Romeo and Juliet. Do those fictional characters exist? If no, how can a sentence containing some fictional names, e.g. 'Romeo loves Juliet', be true?

8. (5%) Let φ and ψ be formulae of the language $\mathcal{L}_K$ suitable for the propositional calculus. Show that if φ ⊨ψ, and yet φ and ψ have no sentence letters in common then either φ is inconsistent or ψ is a tautology.

9. (5%) Frege presented the propositional logic as an axiom system but later Gentzen proposed that the propositional logic can be presented as a system of natural deduction. Briefly describe the difference between these two types of logical systems. Gentzen claimed that the natural deduction he proposed is logically equivalent to the logical system that Frege presented. In what sense two logical systems can be said to be equivalent?

知識論

以下兩題皆須回答，每題分別佔總分之 25%：
(請以數字標明各小題的回答，字跡請力求清晰。)

1. 請說明(1)何謂「可錯論」(fallibilism)。並請說明(2)這一個知識論立場與科學哲學，尤其是科學方法論(scientific methodology)之關係。

2. 我們能否獲得關於過去的知識(knowledge about the past)，例如歷史知識(historical knowledge)？ (1) 關於過去的知識涉及到哪些知識論問題？為什麼？ (2) 根據前一小題的作答，我們能否擁有關於過去的知識？甲認為可以，乙認為不行。你認為哪一種立場比較恰當？請提出論證來支持你的立場。(3) 請舉出一個論證和例子來反對你的立場，並予以回應。

試題隨卷繳回