1. (10 points)

   (a) What's the minimum number of people such that there must be two people born on the same day of the week? Explain your answer.

   (b) What's the minimum number of people such that the probability that two of them were born on the same day of the week is at least 80 percent? Explain your answer.

2. (15 points) Prove that

$$\sum_{k=0}^{n-1} C(n,k) \cdot 2^k = 3^n - 2^n,$$

where $C(n,k)$ is the coefficient of the x^k term in the expansion of $(1+x)^n$

3. (15 points) Solve the following recurrence:

$$a_0 = 2, \quad (1)$$
$$a_1 = 5, \quad (2)$$
$$a_n = 3a_{n-1} - 2a_{n-2} - 2, \text{ for } n \geq 2. \quad (3)$$

4. (20 points) Let R_1 and R_2 be symmetric and transitive relations.

   (a) Prove or disprove $R_1 \bigcup R_2$ is symmetric.

   (b) Prove or disprove $R_1 \bigcup R_2$ is transitive.

5. (20 points) Prove that if each of the 15 edges of the complete graph K_6 is colored either blue or red, there must be at least two monochromatic triangles, where a monochromatic triangle is a triangle with all edges having the same color. Proving that one monochromatic triangle exists gives partial credit.

6. (20 points) Let G be a simple graph with 11 or more vertices. Use Euler's formula to prove that either G or its complementary graph $\bar{G}$ is not a planar graph. Recall that the definition of the complementary graph is the following:

   The complementary graph $\bar{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G.

試題隨卷繳回