

1. Please solve the following non-homogeneous ODE. (15%)

$$y''' - 3y'' + 3y' - y = e^x - x - 1$$

2. Please solve the linear ODE (20%)

$$y'' + 3y' + 2y = r(t), \text{ where } r(t) = \begin{cases} 0 & 0 < t \leq 1 \\ 1 & 1 < t \leq 2 \\ 0 & t > 2 \end{cases}$$

$$y(0) = 0, y'(0) = 0$$

3. (a) Solve the linear system  $Ax = b$ ,

$$\text{where } A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 0 & 10 & 25 \\ 20 & 10 & 0 \end{bmatrix}, \text{ and } b = \begin{bmatrix} 0 \\ 0 \\ 90 \\ 80 \end{bmatrix} \quad (10\%)$$

(b) Describe the way to assess if the solution of this linear system exists and is unique (5%)

4. Please find the Fourier series of the following functions, and show the first three partial sums of the series. (Assumed  $2\pi$  period) (20%)

(1)  $f(x) = x^2 \quad (-\pi < x < \pi)$

(2)  $f(x) = x \quad (-\pi < x < \pi)$

5. Please solve the following boundary value problem using separation of variables. (Show the first three partial sums) (30%)

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \quad \text{for } 0 < x < 2, \quad t > 0,$$

$$y(0, t) = y(2, t) = 0 \quad \text{for } t \geq 0,$$

$$y(x, 0) = 0, \quad \frac{\partial y}{\partial t}(x, 0) = \begin{cases} 2x \\ 0 \end{cases} \quad \text{for } \begin{cases} 0 \leq x \leq 1 \\ 1 < x < 2 \end{cases}$$

試題隨卷繳回