

Note: You can use either Chinese or English to answer the questions below. The total number of points is 100. Each problem is worth 20 points. You need to show all the works to receive the points.

1. The random variables X_1 and X_2 are independent of each other and both have p.d.f. $f(x) = e^{-x}$, $0 < x < \infty$. If $Y_1 = X_1 - X_2$ and $Y_2 = X_1 + X_2$, then please find p.d.f. of Y_1 and p.d.f. of Y_2 .
2. Let Z_1 , Z_2 , and Z_3 have independent standard normal distribution $N(0, 1)$.
 - (a) Find the distribution of $W = \frac{Z_1}{\sqrt{(Z_2^2 + Z_3^2)/2}}$.
 - (b) Find the distribution of $V = \frac{Z_1}{\sqrt{(Z_1^2 + Z_2^2)/2}}$.
3. Let X_1, \dots, X_n be a random sample from the uniform $(\theta, \theta+1)$ distribution. To test $H_0: \theta = 0$ versus $H_1: \theta > 0$, we use the test: reject H_0 if $Y_n \geq 1$ or $Y_1 \geq k$, where k is a constant, $Y_1 = \min\{X_1, \dots, X_n\}$, and $Y_n = \max\{X_1, \dots, X_n\}$.
 - (a) Determine k so that the test will have size α .
 - (b) Find an expression for the power function of the test in part (a).
4. Let X_1, \dots, X_n be a random sample from a gamma distribution with a known parameter $\alpha = 3$ and an unknown parameter $\beta > 0$. Describe the construction of a confidence interval for β .
Hint: the p.d.f. of gamma distribution, $f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$, $0 < x < \infty$.
5. Let S^2 be the sample variance of a random sample of size $n > 1$ from a normal distribution $N(\mu, \theta)$ with mean μ and variance θ , where μ is known.
 - (a) Find the maximum likelihood estimate (MLE) $\hat{\theta}$ of θ .
 - (b) What is the efficiency of S^2 ?

試題隨卷繳回