1. (35%) Consider the following linear programming problem.

$$
\begin{array}{llrrrrcrl}
\text{Maximize} & z = & 10x_1 & -57x_2 & -9x_3 & -24x_4 & & & \\
\text{subject to} & & 0.5x_1 & -5.5x_2 & -2.5x_3 & +9x_4 & \le & 0 & (1) \\
& & 0.5x_1 & -1.5x_2 & -0.5x_3 & +x_4 & \le & 0 & (2) \\
& & x_1 & & & & \le & 1 & (3) \\
& & & & & x_1, x_2, x_3, x_4 & \ge & 0 &
\end{array}
$$

Let x_5, x_6, x_7 be the slack variables for constraints (1), (2), and (3), respectfully. That is, we shall define the initial feasible solution as follows:

$$
\begin{array}{lrrrrr}
x_5 = & & -0.5x_1 & +5.5x_2 & +2.5x_3 & -9x_4 \\
x_6 = & & -0.5x_1 & +1.5x_2 & +0.5x_3 & -x_4 \\
x_7 = & 1 & -x_1 & & & \\
z = & & 10x_1 & -57x_2 & -9x_3 & -24x_4 .
\end{array}
$$

(a) Let us use the simplex method to construct the next iteration. We agree on the following pivoting rule.

- The entering variable will always be the nonbasic variable that has the largest coefficient in the z-row.
- If two or more basic variables compete for leaving the basis, the candidate with smallest subscript will be made to leave.

The basic variables in terms of nonbasic variable after the first iteration are shown as below. Please fill in the following blanks. (Please write your answer in the answer sheet.) (10%)

$$
\begin{array}{llllll}
x_1 = & (\quad)+ & (\quad)x_2 + & (\quad)+x_3 & (\quad)x_4 + & (\quad)x_5 \\
x_6 = & (\quad)+ & (\quad)x_2 + & (\quad)+x_3 & (\quad)x_4 + & (\quad)x_5 \\
x_7 = & (\quad)+ & (\quad)x_2 + & (\quad)+x_3 & (\quad)x_4 + & (\quad)x_5 \\
z = & (\quad)+ & (\quad)x_2 + & (\quad)+x_3 & (\quad)x_4 + & (\quad)x_5
\end{array}
$$

(b) After the sixth iteration, we have

$$
\begin{array}{lrrrrr}
x_5 = & & -0.5x_1 & +5.5x_2 & +2.5x_3 & -9x_4 \\
x_6 = & & -0.5x_1 & +1.5x_2 & +0.5x_3 & -x_4 \\
x_7 = & 1 & -x_1 & & & \\
z = & & 10x_1 & -57x_2 & -9x_3 & -24x_4 .
\end{array}
$$

Please write the basic variables (including z-row) in terms of nonbasic variables after the seventh iteration. (Hint: you may compare the basic variables after the sixth iteration to those in the initial feasible solution.) (10%)

(c) What is the pitfall in Question (b)? (5%)

(i) The optimal solution is unbounded.

(ii) The simplex method cycles.

(iii) The optimal solution does not exist.

(iv) We cannot find the initial feasible solution.

(d) Now let us change the pivoting rule as follows:

- Choose the entering variable as the first variable from the list for which the coefficients in the z-row are positive. (i.e., the entering variable is with the smallest subscript among all variables whose coefficients in the z-row are positive.)
- Among all the potential leaving variables that give the minimum ratio in the ratio test, choose the one with the smallest subscript as the leaving variable.

Solve this liner programming problem by using the pivoting rule in (d) starting from the basic variables, x_5, x_4, x_7, given below. (10%)

$$
\begin{array}{lrrrrr}
x_5 = & & 9x_6 & +4x_1 & -8x_2 & -2x_3 \\
x_4 = & & -x_6 & -0.5x_1 & +1.5x_2 & +0.5x_3 \\
x_7 = & 1 & & -x_1 & & \\
z = & & 24x_6 & +22x_1 & -93x_2 & -21x_3 .
\end{array}
$$

2. (15%) Suppose that your farm is surrounded by $-x+y\le 2$, $2x+3y\le 11$, $x\ge 0$, and $y\ge 0$ on *x-y* plane and your house is located at point $(x,y)=(3/2,5)$.

(a) Please draw the area of your farm on *x-y* plane. (4%)

(b) Suppose that you would like to build a shortest direct route from your house to the farm. The construction cost is the square of the distance of the route you build. Please formulate an optimization model to find out the point (x^*,y^*) in your farm linking to your house at $(3/2,5)$ such that the construction cost of building the shortest direct route from the farm to your house is minimized. (6%)

(c) Solve the problem geometrically. (5%)

3. (25%) *OR-pad* is a new tablet computer on the market. *NTU* is a store that sells *OR-pads*. At the beginning of each day, *NTU* observes its inventory level. After observing the inventory level, an order may be placed (and is immediately received). The following inventory policy is used by *NTU*:

Inventory level at the beginning of a day	Ordering quantity
0	2
1	0
2	0

(In other words, no order will be placed unless inventory level is 0 at the beginning of a day. When the inventory level is 0 at the beginning of a day, *NTU* will order 2 *OR-pads* from its supplier.)

For *NTU*, the daily demands for *OR-pads* are independent and identically distributed (i.i.d.) random variables that have the following probability mass function:

Number of *OR-pads* demanded	Probability
0	0.3
1	0.4
2	0.3

When the demand for *OR-pad* is higher than the inventory level, unmet demand will be lost and no backorders are allowed. (For example, if inventory level is 1 and demand is 2, only 1 *OR-pad* will be sold on that day.)

Let X_n be the inventory level at the beginning day n. **Given $X_0 = 2$**, answer the following questions:

(a) Find the probability mass function of X_1. (5%)

(b) Show that $\{X_0, X_1, X_2, ...\}$ can be modeled as a Markov chain. Define the one-step transition probability matrix of this Markov chain. (5%)

(c) Let π_i be the limiting probabilities (steady-state probabilities) of this Markov chain. In which, $\pi_i = lim_{n\to\infty} P(X_n = i|X_0 = 2)$. Find π_i for all $i \in (0,1,2)$. (5%)

(d) Given $X_n = 1$, find the expected number of *OR-pads* sold by *NTU* on day n. (5%)

(e) Find the long-run average number of *OR-pads* sold by *NTU* per day. (5%)

4. (25%) *RECA* is a store that sells laptop computers to National Taiwan University students. At the beginning of day 1, three laptop computers are available for sale. Because of the coming Lunar New Year holiday, no inventory replenishment is allowed.

The laptop computer can be sold for \$70 or \$60 on the market. Under each of the price settings, the conditional probability distribution of daily demand is listed below:

P(demand = 2 | price = \$70) = 0.3 ; *P(demand = 1 | price = \$70) = 0.7*

P(demand = 2 | price = \$60) = 0.7 ; *P(demand = 1 | price = \$60) = 0.3*

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Moreover, because of the Lunar New Year holiday, any unsold laptop computers will be returned to the manufacturer at the beginning of day 3 for $50 each.

As a store manager, your objective is to maximize the total expected revenue before the Lunar New Year holiday. (Note that the total revenue includes: (1) *revenue from selling the laptop computers in day 1 and day 2*, and (2) *revenue from returning unsold laptop computers to the manufacturer in day 3*.)

(a) Model this dynamic pricing problem by dynamic programming. Clearly define *state variables, action variables, transition probabilities,* and *profit functions* in your dynamic programming model. (10%)

(b) If 2 laptop computers are available for sale at the beginning of day 2, what is the optimal price for day 2? (5%)
(Hint: You may want to solve the dynamic pricing problem by backward induction.)

(c) What is the optimal price for day 1? (5%)

(d) Find is the optimal total expected revenue. (5%)

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