國立臺灣大學101學年度碩士班招生考試試題

1. (25 %) Consider the motion of a satellite with mass m moving in Earth's central gravitational field in R^3. Let the position vector of the satellite from the center of the Earth be denoted by $\mathbf{r} \in R^3$. The force exerted on the satellite, modeled as a particle, is then

$$\mathbf{F} = -\frac{\mu m}{r^3}\mathbf{r},$$

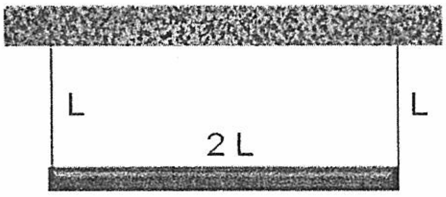
where $r = |\mathbf{r}|$, the length of $\mathbf{r}$, and μ is the gravitational constant of the Earth field.

   (a) Write down the equation of motion of the satellite.
   (b) Show that the motion of the satellite is on a plane.
   (c) Show that the radius vector of the satellite sweeps out equal areas in equal times.
   (d) Show that the orbit of the satellite is an ellipse.
   (e) On the elliptical orbit of the satellite, the point closest to the Earth is called the perigee, and the point which is the most far away from the Earth is termed the apogee. Show that the velocity of the satellite is largest at the perigee, and is the smallest at the apogee.

2. (25 %) Consider the motion of a particle of mass m moving near the surface of the Earth, which is rotating about the axis of the north pole by angular velocity ω_e. Let the velocity of the particle relative to the Earth be denoted by $\mathbf{v}_r$.

   (a) Write down the Coriolis force.
   (b) Show that the particle will be deflected to the right-hand side of its moving direction as the effect of the Coriolis force.
   (c) Explain the reason why the outlook of a typhoon in the Northern Hemisphere is counter-clockwise.

3. (25 %) Consider a slim rigid bar of mass m and length $2L$ under the constant gravitational field g as shown in the plot. Essentially all the bar's mass is concentrated so you can locate the mass center at the middle point of the bar. The two ends of the horizontal bar are supported by two massless inextensible strings of length L. (a) If you pull the bar to the right so that the supporting strings are at 45 degree from their equilibrium (vertical) lines and then release it with zero velocity. Find the tension of the strings at the instant of release. (b) Determine the equation of motion for the bar. (c) Find the motion period for small oscillation of the bar.



4. (25 %) Consider a homogeneous disk of mass m and radius R rolling down a rough inclined wire due to the constant gravitational field g under the Coulomb friction with coefficient μ. The inclined wire is angled at α with respect to the horizontal line as shown in the plot. (a) Derive the rotational moment of inertia about the disk's mass center. (b) Determine the minimum μ for which the disk rolls without slip down the inclined wire. (c) Write down the equations of motion under the condition described in (d) and find the solutions.