1. (10%) Find all eigenvalues of,

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

2. (10%) Find all values of h so that the following vectors are linearly independent,

$$x = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \quad y = \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \quad z = \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$$

3. (15%) Find the general solution to the following linear system of equations,

$$X_2 + 2X_3 = 0$$

$$4X_1 + 5X_2 + 6X_3 = 0$$

$$8X_1 + 9X_2 + 10X_3 = 0$$

4. (15%) Mark each statement as either true or false.
   (a) Suppose that $\mathbf{Ax} = \mathbf{b}$, where $A \in R^{n \times n}$, has no solutions. The corresponding homogeneous system, $\mathbf{Ax} = \mathbf{0}$, has only the trivial, $\mathbf{x} = \mathbf{0}$, solution.
   (b) If $\mathbf{A} \in R^{m \times n}$ has a row of zeros then $\mathbf{Ax} = \mathbf{0}$ always has infinitely-many solutions.
   (c) The system $\mathbf{Ax} = \mathbf{0}$, where $\mathbf{A} \in \mathbf{R}^{3 \times 4}$ has only the trivial solution.

5. (15%) Solve the following initial value problem.

$$y_1' = 2y_1 - y_2$$

$$y_2' = -y_1 + y_2$$

$$y_1(0) = 1, \quad y_2(0) = 0$$

6. (15%) Draw the graph of the even and odd extensions of the function (include several periods).

$g(x) = x^3$ on $0 \le x \le 1$.

7. (20%) Show how to solve the following differential equation. You will need to use separation of variables. Explain all your steps.

$$u_t = c^2 u_{xx} - \alpha u \qquad 0 \le x \le L, t \ge 0$$

$$u(0,t) = 0 \qquad t \ge 0$$

$$u(L,t) = 0 \qquad t \ge 0$$

$$u(x,0) = f(x) \qquad 0 \le x \le L$$

試題隨卷繳回