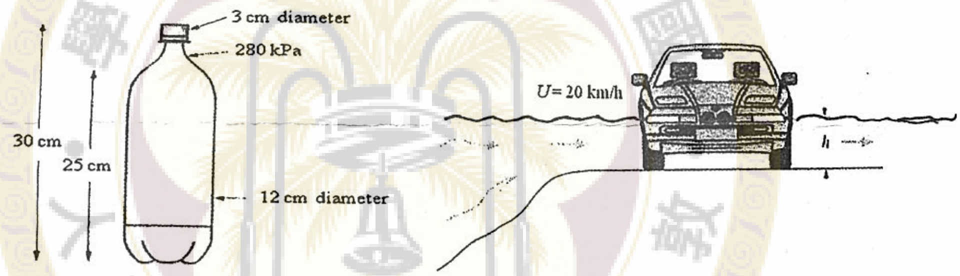
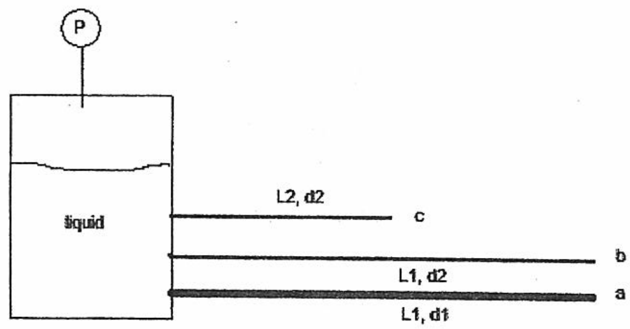
試題共六大題，總分 100 分

1. The air pressure in the top of the 2-liter pop bottle shown in the following figure is 280 kPa, and the pop depth is 25 cm. The bottom of the bottle has an irregular shape with diameter of 12 cm. If the bottle cap has a diameter of 3 cm what is the magnitude of the axial force require to hold the cap in place? Please also determine the force needed to secure the bottom 5 cm. of the bottle to its cylindrical sides. Assume that the effect of the weight of the pop is negligible. **(15 points)**

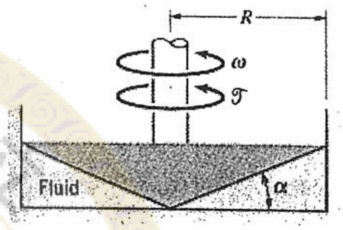
2. During a flash flood, water rushes over a road as shown in the following figure with a speed of 20 km/h. How do you estimate the maximum water depth, h, that would allow a car to pass without being swept away. List all assumptions and show all calculations. **(15 points)**



3. Fluid flow in micro-channels (Hagen-Poiseuille flow) **(15 points)**
在微小管流中，由於管徑小，在不是很快的流速下，其雷洛數(Re)均不會很大。實驗中驗證，在平滑管中，當 Re < 1500 時，其阻力係數 λ = constant/Re，而其中之 $\lambda = (2\Delta p/\rho u^2)\ (d/L)$；而 u: 流體平均流速，d: 管直徑，L: 管長度，Δp: 壓差。
   (1) 推導λ式中之常數值；(9%)
   (2) 下圖中管 a 與 管 b 之流量比；(3%)
   (3) 下圖中管 a 與 管 c 之流量比。(3%)



4. A cone and plate viscometer consists of a cone with a very small angle α which rotates above a flat surface as shown in the following left figure. The torque, $\mathcal{T}$, required to rotate the cone at an angular velocity, ω, is a function of the radius, R, the cone angle, α, and the fluid viscosity, μ, in addition to ω. With the aid of dimensional analysis, determine how the torque will change if both the viscosity and angular velocity are doubled. **(15 points)**



5. What is (are) the basic assumption(s) for the boundary layer approximation in external flow field? What are the differences between the Navier-Stokes equations and the boundary layer equations? **(5 points)**

6. Consider a circular cylinder of radius a in a free stream with velocity U and pressure p_0. Suppose you are given velocity potentials (ϕ) and stream functions (ψ) as follows:
   i) Uniform flow at angle α with the x axis: $\phi = U(x\cos\alpha + y\sin\alpha)$, $\psi = U(y\cos\alpha - x\sin\alpha)$.
   ii) Doublet: $\phi = K\cos\theta / r$, $\psi = -K\sin\theta / r$, where K is the strength.
   iii) Free vortex: $\phi = \Gamma\theta / (2\pi)$, $\psi = -\Gamma \ln r / (2\pi)$, where Γ is the circulation.
   iv) Source ($m > 0$) or sink ($m < 0$): $\phi = m \ln r / (2\pi)$, $\psi = m\theta / (2\pi)$, where m is the volume rate of flow emanating from the line (per unit length).

(a) Derive the velocity distribution around the circular cylinder with the assumption of negligible viscosity. What are the resultant forces (per unit length) developed on the cylinder respectively in x and y directions? This leads to the well known D'Alembert's paradox. Please interpret it and explain the reason. **(15 points)**

(b) Now consider the effect of viscosity. If the viscous force on the fluid is relatively larger than the inertial force, how would you proceed to solve the flow field for distributions of velocity and pressure around the circular cylinder? If you cannot obtain the final solutions immediately, just lay out the procedure as clear as possible, e.g., write down the governing equations you may need with adequate boundary conditions and assumptions. (Hint: It would be helpful by considering expressions in terms of dimensionless parameters.) Plot the variation of the pressure coefficient, $C_p = 2(p - p_0)/(\rho U^2)$ with respect to the angle along the cylinder surface, θ, from 0 to π. Compare the results, with and without turbulence, to that in an inviscid flow with clear illustrations. Thus elucidate the mechanism of surface roughness on a golf ball. **(15 points)**

(c) If the flow speed increases from 0 to 1000 m/s at the standard atmospheric condition, how is the drag coefficient varying? Interpret the characteristic structure and mechanisms that cause these essential changes. **(5 points)**