國立臺灣大學101學年度碩士班招生考試試題

Problem (1) (25%)

As shown in Fig. 1a, water flows from a big reservoir through a gate and into a stream. Both far upstream (with flow depth H) and far downstream (with flow depth h), the flow condition is steady uniform.

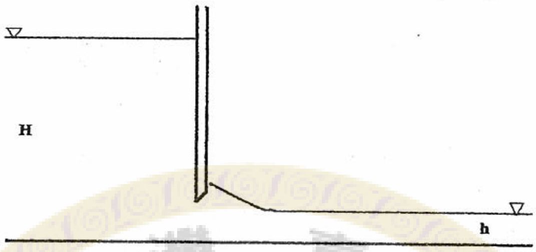

Fig. 1a

(a) (10%).If the system is adiabatic, has no shaft energy input and no friction loss but do suffer energy loss when going through the gate with the energy head loss expressed as $h_L = 5\frac{v^2}{2g}$, what are the relation between H, and h.

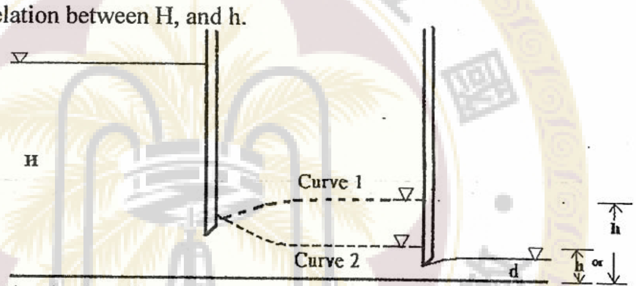

Fig. 1b

(b) (5%) Now, as depicted in Fig.1b, there is another gate far downstream. If we lower the gate so that the gap is less than the downstream steady flow depth, what will happen to the flow? Describe the changes and then explain which of the two curves (curve 1 and curve 2) in Fig. 1b maybe the better possible final profile when the steady state reached again.

(c) (10%) If h is the depth just behind the second gate, using the same assumption as in (a), calculate the relation between H, h and d.

Problem (2) (25%) In plain Cartesian Coordinate (x,y), a Stream function $\varphi = x^2 y$ is given

(1) (5%) Is this an irrotational flow field? Calculate vorticity and then prove your answer.

(2) (5%) Prove that x and y axis are both streamlines

(3) (5%) Calculate the acceleration for this flow field

(4) (5%) If y axis is a wall and pressure is P at the origin, derive the pressure distribution along y axis.

(5) (5%) Streamlines cannot cross. However, there are streamlines meet at the origin. Does this imply that origin is a singular point? If your answer is YES, then explain why there is such a singular point. If your answer is NO, explain why streamlines intersect.

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Problem (3) (25%) A rectangular tube connected to a quarter circular tube as shown in Fig. 3.

(a) (10%) Water is stationary (no hole) in the tube. Calculate the total force on water along surface ABCD. Give the magnitude and direction of the total force.

(b) (5%) If this system is put on a frictionless table and there is a small hole at the midpoint of CD. The round hole has diameter 1cm. Since the hole is so small, the velocity within the system can be neglected. What will be the free surface become?

(c) (10%) If the table surface has large enough friction so the tube will not move, calculate the total force on surface ABCD and CDEF. Give the magnitude and direction of the total force.

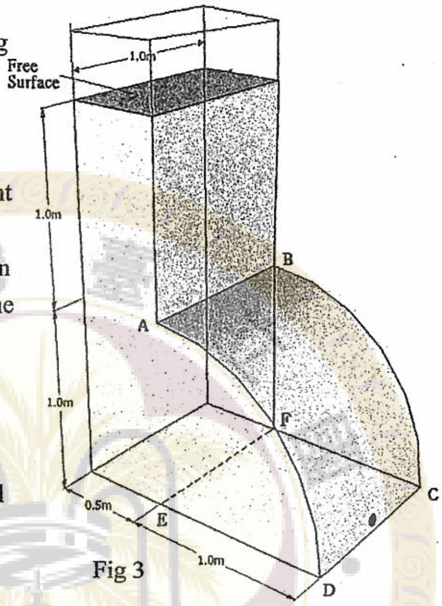


Fig 3

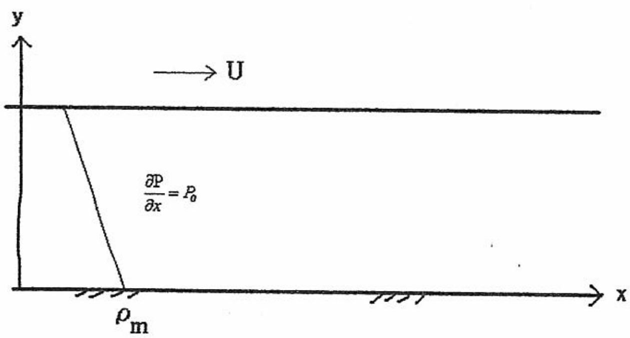
Problem (4) (25%) Newtonian fluid flows within two horizontal plates. The lower plate is stationary and cannot be moved. The upper plate is pulled towards right with speed U. There is a positive constant pressure gradient $\frac{\partial P}{\partial x} = P_0 > 0$ in the channel. The density is of the fluid is a function of y $\rho = \rho_m - cy$ where ρ_m is the maximum density at the bed.

(a) (5%) Write down the complete Navier-Stokes equations. Note kinematic viscosity does not change with density. Then write down all the necessary and proper initial or boundary conditions.

(b) (5%) For steady flow, first derive the pressure distribution within the fluid.

(c) (10%) Then derive the velocity profile for u and v

(d) (5%) Is the net flow rate can be zero? If your answer is YES, write the condition for it. If your answer is NO, write down the explanation.



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