- 本試題共 6 大題, 合計 100 分。
- 請依題號依序作答。
- 請詳述理由或計算推導過程, 否則不予計分。

1. (20%)

   (a) Suppose $Z = XY$, where X and Y are independent random variables. Write $Var(Z)$ in terms of $Var(X)$, $Var(Y)$, $E(X)$ and $E(Y)$.

   (b) Suppose that $Y = Z - X$ is independent of Z and of X. Show that Y is a constant.

2. (15%) Suppose that $X \sim f(x)$. Let $Y = F(X)$ where $f(.)$ is the probability density function (pdf) of X and $F(.)$ is the cumulative distribution function (cdf) of X. Find the distribution of Y and $E(Y)$.

3. (15%) You are interested in estimating $\theta = \mu_1 - \mu_2$, where $X_1 \sim N(\mu_1, 50)$ and $X_2 \sim N(\mu_2, 100)$. Assume that X_1 and X_2 are independent. You can afford a total of 100 observations. Determine how many you should draw on X_1 and how many on X_2 and Why.

4. (15%) The following wage equation was run using 8,654 individuals with either high school or college education.

   $$\widehat{\log(wage)} = 10.17 + 0.27\,male + 0.36\,college - 0.08\,male \times\ college, \quad R^2 = 0.1296$$

   where *college* is a dummy variable: $college = 1$ if college graduates, $college = 0$ if high school graduates; *male* is a dummy variable: $male = 1$ if male and $male = 0$ if female.

   (a) Interprete the regression coefficient of $male \times college$?

   (b) If we define another female dummy variable where $female = 1$ if female, $female = 0$ if male, and run the following regression:

   $$\widehat{\log(wage)} = \hat{\beta}_0 + \hat{\beta}_1\ female + \hat{\beta}_2\ college + \hat{\beta}_3\ female \times\ college$$

   What are the regression coefficients $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$ and $\hat{\beta}_3$?

   (c) Is the R^2 in (b) greater than, less than or equal to 0.1296? Why?

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5. (15%) The model is

$$Y_i = \beta X_i + u_i, \quad E(u_i|X_i) = 0, \quad E(u_i^2|X_i) = \sigma^2 X_i^2$$

where X_i is a scalar. Consider two estimators

$$\hat{\beta} = \frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2} \quad \text{and} \quad \tilde{\beta} = \frac{1}{n}\sum_{i=1}^n \frac{Y_i}{X_i}.$$

(a) Are $\hat{\beta}$ and $\tilde{\beta}$ consistent for β? Why?

(b) Find $E(\tilde{\beta}|X)$ and $Var(\tilde{\beta}|X)$.

(c) Is $\hat{\beta}$ more efficient than $\tilde{\beta}$? Why?

6. (20%) A researcher considers two regression specifications:

$$\log Y_i = \beta_1 + \beta_2 \log X_i + u_i \quad (1)$$

$$\log \frac{Y_i}{X_i} = \alpha_1 + \alpha_2 \log X_i + v_i \quad (2)$$

where u_i and v_i are error terms.

Writing $y_i = \log Y_i$, $x_i = \log X_i$, and $z_i = \log \frac{Y_i}{X_i}$, and using the sample of n obsevations, the researcher fits the two specifications using OLS,

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i \quad (3)$$

$$\hat{z}_i = \hat{\alpha}_1 + \hat{\alpha}_2 x_i \quad (4)$$

(a) Using the expressions for the OLS regression coefficients, what is the relationship between $\hat{\beta}_2$ and $\hat{\alpha}_2$?

(b) What is the relationship between $\hat{\beta}_1$ and $\hat{\alpha}_1$?

(c) Show that the relationship between the fitted values of y, the fitted values of z, and the actual values of x is $\hat{y}_i - x_i = \hat{z}_i$.

(d) Compare the residuals for regression (3) and (4). Are they identical? Why?

(e) Whether R^2 would be the same for the two regressions? Why?

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