國立臺灣大學101學年度碩士班招生考試試題

※ 注意：請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。

I. Fill in the blanks:

1. {15 points} Which of the following statements are true? Answer: ________.

   (a) If $\vec{x}$ is a nonzero vector and $\vec{0}$ the zero vector in $\mathbb{R}^n$ such that $A\vec{x} = \vec{0}$, then the determinant of the $n \times n$ matrix A is vanishing.

   (b) The set of vectors $\{(1,2,4)^T, (2,1,3)^T, (4,-1,1)^T\}$ span the vector space $\mathbb{R}^3$.

   (c) If the vectors x_1, x_2, ..., x_n span a vector space $\mathbb{V}$, then they are linearly independent.

   (d) If S and T are subspaces of a vector space $\mathbb{V}$, then the union $S \cup T$ is also a subspace of $\mathbb{V}$.

   (e) If the $n \times n$ matrix $A \neq O$ (where O denotes the zero matrix) but $A^k = O$ for some positive integer k, then A must be singular.

2. {15 points} Using LU factorization, the matrix A can be written as

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 0 & 2 & 1 \\ 2 & 3 & 1 & 3 \\ 1 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 1/4 & 1 & 0 \\ 1 & 1/2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -4 & 0 & -3 \\ 0 & 0 & -1 & -1/4 \\ 0 & 0 & 0 & 1/2 \end{pmatrix} \equiv LU.$$

   (a) The determinant $\det(A) =$ ________.

   (b) The solution of $A\vec{x} = (1,1,1,1)^T$ is $\vec{x} = (x_1, x_2, x_3, x_4)^T =$ ________. Note that here the superscript T denotes transpose.

   (c) The third column of A^{-1} (expressed as a column vector) = ________.

3. {10 points} Consider the 2×2 matrix

$$A = \begin{pmatrix} a & 1 \\ \gamma & \delta \end{pmatrix}.$$

   (a) To make A possess eigenvalues $\lambda = 1$ and $\lambda = -1$ for any given value of a, one must choose γ and $\delta =$ ________.

   (b) To make the eigenvectors corresponding to $\lambda = 1$ and $\lambda = -1$ orthogonal, a must be chosen to be ________.

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4. {10 points} Let

$$\vec{b}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{b}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \vec{b}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

and let L be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ defined by

$$L(\vec{x}) = x_1\vec{b}_1 + x_2\vec{b}_2 + (x_1 + x_2)\vec{b}_3 \qquad \forall\, \vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2.$$

If A is the matrix representing L with respect to the bases $\{\vec{e}_1, \vec{e}_2\}$ and $\{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$, where

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

then $A =$ __________.

II. Calculation problems:

5. {10 points}With $y = y(x)$ being a function of the independent variable x, solve the differential equation

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 3e^x.$$

6. {20 points}With $y = y(x)$ being a function of the independent variable x, solve the differential equation

$$\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} + (16x^2 + 4 + \frac{1}{4x^2})y = 4e^{-2x^2}.$$

7. {20 points}With $y = y(x)$ being a function of the independent variable x, solve the differential equation

$$4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 1.$$

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