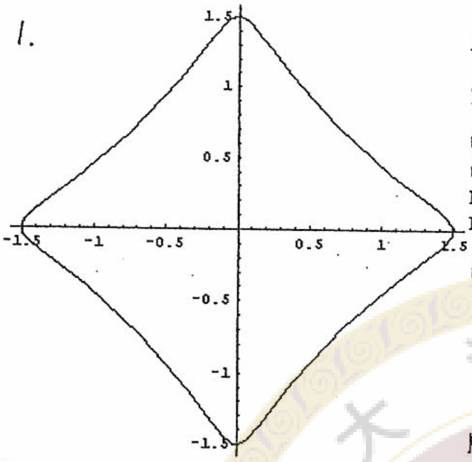
國立臺灣大學101學年度碩士班招生考試試題

1.



Mathematica® 4.0

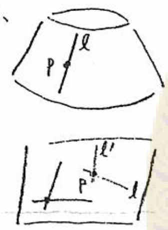
$x(\theta)=(1+\frac{1}{n})\cos\theta+\frac{1}{n}\cos((n-1)\theta)$

$y(\theta)=(1+\frac{1}{n})\sin\theta-\frac{1}{n}\sin((n-1)\theta)$

$0\leqq\theta\leqq2\pi$ is not the parametric equation of a hypo-cycloid. For $n=3$, is it a convex curve? For $n=4$, is it a convex curve?

(25/100)

2、



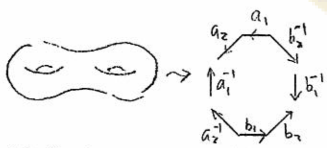
Let R be a ruled surface. Through each point $p\varepsilon R$, there is a segment l of straight line $p\varepsilon l\subseteq R$. Can you prove the Gauss curvature K of R vanishes identically $K\equiv0$? If in addition there is another segment $l'\neq l$ through each point p, $p\varepsilon l\cap l'$, $l\cup l'\subseteq R$, can you prove R is contained in a plane? (25/100)

3、Let ω^1 and ω^2 be linearly independent in a neighborhood U of the origin, $\omega^1=P(x,y)dx+Q(x,y)dy$, $\omega^2=R(x,y)dx+S(x,y)dy$, $\det=\begin{vmatrix}P&Q\\R&S\end{vmatrix}\neq0$. Can you find a differential $\omega_1^2=-\omega_2^1$ so that the exterior derivatives and exterior products satisfy

$d\omega^1=\omega^2\wedge\omega_2^1$

$d\omega^2=\omega^1\wedge\omega_1^2$? If yes, $\omega_1^2=?dx+?dy$. (25/100)

4.



Let G_2 be the fundamental group of a surface of genus 2. G_2 is generated by a_1, a_2, b_1, b_2 satisfying the relation $a_1a_2a_1^{-1}a_2^{-1}b_1b_2b_1^{-1}b_2^{-1}=1$. If G_3 is generated by x_1, x_2, y_1, y_2, z_1, z_2 satisfying the relation $x_1x_2x_1^{-1}x_2^{-1}y_1y_2y_1^{-1}y_2^{-1}z_1z_2z_1^{-1}z_2^{-1}=1$, can you find a subgroup of G_2 isomorphic to G_3? (25/100)

試題隨卷繳回