

## 第一大題 1~5 選擇題(包括單選題與複選題)

Problems (1-5) are mixed with single-choice and multiple-choice problems.

※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

1. (10%) The Laplace transform,  $Y(s) = \mathcal{L}\{y(t)\}$ , can be used to solve the linear differential equations with variable monomial coefficients,  $2y'' + ty' - 2y = 10, y(0) = y'(0) = 0$ . Please solve  $Y(s)$ , and then find  $y(t) = \mathcal{L}^{-1}\{Y(s)\}$ ,

(A)  $y(t) = 5t^2$ , (B)  $Y(s) = \frac{5}{s^3}$ , (C)  $Y(s) = \frac{e^{s^2}}{s^3}$ , (D)  $y(t) = \frac{t^2}{2}$ , (E)  $Y(s) = \frac{5}{s^2}$ ,

2. (10%) After a mass  $m$  is attached to an 'aging' spring,  $K(t) = ke^{-at}$ , the free un-damped motion equation can be transform into a different equation of  $P(s)x'' + Q(s)x' + s^2x = 0$  with the change of variables,  $s = f(t)$ . Find  $x(t)$ .

(A)  $P(s) = s^2$ , (B)  $Q(s) = 1$ , (C)  $f(t) = \frac{2}{\alpha} \sqrt{\frac{k}{m}} e^{-at}$ , (D)  $x(t) = c_1 \sin(t) + c_2 \cos(t)$ , (E)  $Q(s) = s$

3. (10%) Solve  $xy'' + (x-6)y' - 3y = 0$ , we obtain two solutions:  $y_1 = 1 - \frac{1}{2}x + Px^2 - \frac{1}{120}x^3 + Qx^4$   
 $y_2 = x^7 - Rx^8 + Sx^9 - \dots$

(A)  $P = \frac{1}{10}$ , (B)  $Q = \frac{1}{360}$ , (C)  $S = \frac{4}{36}$ , (D)  $Q = 0$ , (E)  $R = \frac{1}{2}$

4. (10%)  $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ , and  $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(A)  $e^{At} = \begin{pmatrix} e^{2t} & 0 \\ 0 & e^t \end{pmatrix}$ , (B)  $e^{At} = \begin{pmatrix} e^t & 0 \\ 0 & e^{2t} \end{pmatrix}$ , (C)  $e^{Bt} = \begin{pmatrix} 0 & \cosh t \\ \sinh t & 0 \end{pmatrix}$ ,

(D)  $e^{Bt} = \begin{pmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t \end{pmatrix}$ , (E)  $e^{Bt} = \begin{pmatrix} \sinh t & \cosh t \\ \cosh t & \sinh t \end{pmatrix}$ ,

5. (10%) Use Fourier integral representation of  $f(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 < x < 2 \\ 0, & x > 2 \end{cases}$ , to find  $\int_0^\infty \frac{\sin \alpha}{\alpha} d\alpha = P$

(A)  $P = \frac{\pi}{4}$ , (B)  $P = \frac{2}{\pi}$ , (C)  $P = \frac{\pi}{2}$ , (D)  $f(x) = \frac{\pi}{2} \int_0^\infty \left[ \left( \frac{\sin 2\alpha}{\alpha} \right) \cos \alpha x \right] d\alpha$

見背面

6. Li-Chi is a college student who knows probability very well. He claims that (1) the probability of any event must be positive, and (2) the sum of the probabilities of all events for any given experiment must be 1. Do you agree with his claims? If yes, please simply write "YES" in your answer sheet. Otherwise, please correct his claim(s) (Your answer must be less than 20 words in total in English or Chinese or you will receive ZERO credit) (8%)

7. Li-Chi throws a fair six-sided die continuously. How many 5's will he observe on average before finally getting two 2's? (10%)

8. Let  $X$  and  $Y$  denote the time when the first and second lightings strike in a stormy night. Note that  $X$  and  $Y$  are continuous random variables with a joint PDF  $f_{X,Y}(x,y) = \begin{cases} \beta^2 e^{-\beta y} & 0 \leq x < y \\ 0 & \text{otherwise} \end{cases}$ . Please find (1) the PDF of  $U=Y-X$  (9%), and the conditional PDF  $f_{X|Y}(x|y)$  (8%). (Please specify the range of  $U$ ,  $x$ , and  $y$  in your answers or you will receive ZERO credit.)

9. Li-Chi is playing a video game shown below. His goal is to destroy the space ship using the machine gun. Assume that the ship moves horizontally at a speed of 8 cm/second and it bounces back immediately when reaching each side. The ship starts at the left side at  $t=0$ . Assume that Li-Chi fires once randomly within each second and the "bullet" flies at an infinitely high speed. What is the probability that he can hit the space ship for every firing? (5%) What is the average time (in seconds) it takes to hit the ship? (10%)

