1. Regression methods were used to analyze the data from a study investigating the relationship between roadway surface temperature (x) and pavement deflection (y). Summary quantities were n=20, $\sum y_i = 12.75$, $\sum y_i^2 = 8.86$, $\sum x_i = 1478$, $\sum x_i^2 = 143220$, $\sum x_i y_i = 1085$. (30%)
   (1) Calculate the least squares estimates of the slope and intercept of a simple linear regression model.
   (2) Use the equation of the fitted line to predict what mean pavement deflection would be observed when the surface temperature is $85^{\circ}F$
   (3) Estimate σ^2.

2. The random variable X has a binomial distribution with n=10 and p=0.5. Sketch the probability mass function of X. (10%)
   (1) What value of X is most likely?
   (2) What value(s) of X is (are) least likely?

3. The elapse time X between the occurrences of two consecutive (independent) storm events (commonly known as the inter-arrival-time) can be characterized by a negative exponential distribution with the following probability density function:

$$f_X(x;\lambda) = \lambda e^{-\lambda x}, \quad x \geq 0.$$

where $\lambda > 0$. Suppose that inter-arrival-time (in hours) of a series of storm events are observed and listed below.

150	2	15	2	79	54	60	47	79	15
49	19	70	17	30	24	106	55	27	71
53	53	6	173	59					

   (1) Derive and calculate the maximum likelihood estimator of λ using the above observations? (10%)
   (2) If a storm event just occurred, estimate the probability that the next storm event will occur within 48 hours? (10%)

4. Random variables X and Y are correlated with correlation coefficient $\rho_{XY} = 0.8$. The variances of X and Y are 125 and 400, respectively. Let $Z = 2.5X - 1.2Y$. Calculate the variance of Z. (20%)

5. A factory has ten production lines that produce the same products. These production lines are run under the same process and are independently operated. A new treatment was implemented in the manufacturing process to reduce the rate of defect products (瑕疵率). The following table lists the defect rates (in percentages, %) of individual production lines before and after implementation of the new treatment. (20%)

Production Line	1	2	3	4	5	6	7	8	9	10
Before	**8.4**	**9.5**	**9.7**	**13.0**	**6.8**	**13.0**	**9.7**	**6.4**	**10.7**	**8.1**
After	**8.6**	**6.3**	**8.6**	**5.7**	**7.4**	**6.6**	**6.4**	**7.1**	**4.5**	**8.8**

Assume the defect rates are normally distributed. Conduct a hypothesis test to determine whether the new treatment did help to reduce the mean defect rates. (Level of significance $\alpha = 0.05$)

Degree of freedom	Cumulative probability of t distribution $P[T \le t]$				
n	0.75	0.9	0.95	0.975	0.99
5	0.727	1.476	2.015	2.571	3.365
6	0.718	1.440	1.943	2.447	3.143
7	0.711	1.415	1.895	2.365	2.998
8	0.706	1.397	1.860	2.306	2.896
9	0.703	1.383	1.833	2.262	2.821
10	0.700	1.372	1.812	2.228	2.764

For n = 8, $P[T \le 0.706 = 0.75]$

試題隨卷繳回