

1. Find the moment generating function corresponding to the following p.d.f

$$(a) f(x) = \frac{1}{2\beta} e^{-|x-\alpha|/\beta}, \quad -\infty < x < \infty, \quad -\infty < \alpha < \infty, \quad \beta > 0 \quad (10\%)$$

$$(b) P(X=x) = \binom{r+x-1}{x} p^r (1-p)^x, \quad x=0,1,\dots, \quad 0 < p < 1, \quad r > 0 \text{ an integer} \\ (10\%)$$

2. Suppose X represents a single observation from the following p.d.f

$$f(x) = \theta x^{\theta-1}, \quad 0 < x < 1; \quad f(x) = 0, \text{ otherwise.}$$

Find the most powerful test with significance level $\alpha=0.05$ to test $H_0: \theta=1$ versus $H_a: \theta=2$ (15%)

3. Let X_1, X_2, \dots, X_n be a random sample from

$$f(x) = \theta x^{-2}, \quad 0 < \theta \leq x < \infty.$$

(a) What is a sufficient statistic for θ ? (5%)

(b) Find the maximum likelihood estimator of θ . (5%)

(c) Find the method of moments estimator of θ . (5%)

4. Define: (25%)

(a) random experiment, probability set function and random variable.

(b) method of least squares and method of maximum likelihood.

(c) sufficiency, consistency and efficiency.

(d) type I error, type II error and power function.

(e) uniformly most powerful test and likelihood ratio test.

5. State the Central Limit Theorem. (5%)

6. State the Rao-Blackwell Theorem. (10%)

7. State the Neyman-Pearson Theorem. (10%)

試題隨卷繳回