1. (20%) In the following tableau for the maximization problem, there are six unknown constants d, e, f, g, h, and i. Assume that x_4, x_5, and x_6 are the slack variables, and no artificial variables. Set conditions or restrictions on these constants, so that the following statements in each question are valid.

z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
1	h	i	0	0	3	0	
0	4	d	1	0	e	0	f
0	-1	-5	0	1	-1	0	2
0	g	-3	0	0	-4	1	3

(a) (4%) The current tableau is optimal and an alternative solution exists.
(b) (4%) One of the constraints cannot be satisfied.
(c) (4%) The current solution is a degenerate basic feasible solution.
(d) (4%) The current solution is feasible but the problem is unbounded.
(e) (4%) The current solution is feasible and the objective value can be improved by replacing x_6 by x_1. What will the change in the objective value be after the pivot?

2. (15%) Consider a mixed integer programming problem of maximizing the objective. In the problem, there are four decision variables where x_1, x_2, and x_3 are binary and x_4 is nonnegative. Some possible combinations of these binary variables are shown as in the following table, and **x** and Z denotes the optimal solution and the corresponding objective value by having the integer variables relaxed or fixed. Answer the questions by using the branch-and-bound algorithm based on the given table. Start with Node α and use "best bound first" as the branch rule.

(a) (5%) What is the value of the first lower bound?
(b) (5%) What node(s) can be fathomed when the second feasible solution obtained in the branch-and-bound tree?
(c) (5%) What is the optimal solution?

Node	x_1	x_2	x_3	x (x_1, x_2, x_3, x_4)	Z
α	#	#	#	(0.2, 1, 0, 0)	72.8
A	#	#	1	(0, 0.8, 1, 0)	69.4
B	#	0	#	(0.7, 0, 0, 0)	71.8
C	#	0	1	(0.4, 0, 1, 0)	68.6
D	#	1	1	(0, 1, 1, 0.5)	67.0
E	0	#	#	(0, 1, 0.67, 0)	70.7
F	0	#	0	(0, 1, 0, 2)	28.0
G	0	0	#	infeasible	---
H	1	#	#	(1, 0.3, 0, 0)	64.6
I	1	#	1	(1, 0, 1, 0)	53.0
J	1	0	#	(1, 0, 0, 0)	59.0
K	1	1	#	(1, 1, 0, 0)	52.0
L	1	1	1	(1, 1, 1, 0)	41.0

(#: the integer constraint of that variable is relaxed.)

3. (15 %) The Farm Tractor Company is a manufacturer of tractor parts at City 1, and ships tractor parts from City 1 to City 6 by railroad. Currently, the company has contracted the available railroad cars on each route during a week, shown in the following table. All railroad cars are of equal capacity. Due to the increasing demand at City 6, The Farm Tractor would like to add one more railroad car to the existing network, so that more tractor parts can be shipped from City 1 to City 6. Please formulate a mathematical model to determine which route(s) the extra car should be assigned to? (You don't have to obtain the solution to your model.)

From City	1	1	2	2	2	3	3	4	4	5
To City	2	3	3	4	5	4	6	5	6	6
Available Cars	8	5	7	5	2	3	6	1	3	4

見背面

4. (15%) Answer the following questions

(a) (7%) Let f_1 and f_2 be convex functions on the convex set Ω. Prove that the function $f_1 + f_2$ is convex.

(b) (8%) Let f be a convex function on a convex set Ω. Prove that the set $\theta_c = \{\mathbf{x} : \mathbf{x} \in \Omega, f(\mathbf{x}) \le c\}$ is convex for every real number c.

5. (20%) Consider the problem

Maximize: $f(\mathbf{x}) = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$
Subject to: $2x_1 - x_2 = 4$

(a) (5%) Form the Lagrangian Function to the above maximization problem.

(b) (5%) What is the stationary point for the above maximization problem?

(c) (10%) Is the stationary point you found in (b) a maximum, minimum, or none of the above? Please explain it.

6. (15%) Consider an M/M/1 queue with an infinite capacity in a firm. It is possible to alter the service rate, μ, and that this rate can be any value over the interval from λ to $+\infty$, where λ is the arrival rate. Obviously, a higher μ will entail a higher cost. Assume that the relationship between μ and the cost is linear. We let s denote the marginal cost of increasing μ; in other words, the cost of providing service is $s\mu$. We've known the expected number of units in an M/M/1 queueing system is $L = \lambda/(\mu - \lambda)$. Let w denote the cost of each unit waiting in line. The expected waiting cost per unit of time is that w times the expected number of units in the system, L. (Note: We do not explicitly say "customers" in the question because "customers" in queueing systems may be calling units within the firm.)

(a) (5%) Consider the cost of providing service and the expected waiting cost per unit of time. Formulate the expected-cost equation as a function of μ, $C(\mu)$.

(b) (10%) What is the optimal service rate, μ^*?