

1. Solve the following ordinary differential equations: (20%)

(1) $y'' - y' - 2y = 5 \cos x$

(2) A damped oscillation equation: $my'' + cy' + ky = 0$; discuss the solution,

if (a) $m=1, c=2, k=2$ (b) if $m=1, c=4, k=2$

2. The following is the Lucas series, (15%)

$1, 3, 4, 7, 11, 18, 29, 47, 76, \dots$

$a_0=1, a_1=3, a_2=4, a_3=7, a_4=11, a_5=18, \dots$

(a) Please find all mathematical rules.

(b) Please find $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = ?$

(c) If $F(x) = \frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots$

Please find $1 - 1 + 1 - 1 + 1 - 1 + \dots = ?$

3. Given

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

(a) find eigenvalues and eigenvectors of A . (10%)

(b) find all solutions of $(A - 2I)x = (1 \ 0 \ 0 \ 0)^T$ (10%)

(c) find eigenvalues and eigenvectors of $(A - 2I)^{100}$. (10%)

4. Consider the boundary-value problem $y'' + \lambda y = 0, y(0) = y(2\pi), y'(0) = y'(2\pi)$.

Show that except for the case $\lambda = 0$, there are two independent eigenfunctions corresponding to each eigenvalue. (15%)

5. Find the Taylor series of the following functions about the point $Z = a$ and determine the radius of convergence. (20%)

(1) $\ln Z, a = 1$

(2) $\cos Z^2, a = 0$.

試題隨卷繳回