1. Consider a linear system whose augmented matrix is of the form (15%)

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & a & b \end{array}\right]$$

(a) For what values of a and b will the system have infinitely many solutions?
(b) For what values of a and b will the system be inconsistent?

2. True or false, with reason if true or counterexample if false: (20%)
(a) If A is invertible and its rows are in reverse order in B, then B is invertible.
(b) If A and B are symmetric then AB is symmetric.
(c) If A and B are invertible then BA is invertible.
(d) Every nonsingular matrix can be factored into the product $A = LU$ of a lower triangular L and an upper triangular U.

3. Let $u_1 = (1, 1, 1)^T$, $u_2 = (1, 2, 2)^T$, $u_3 = (2, 3, 4)^T$. (15%)
(a) Find the transition matrix corresponding to the change of basis from $[e_1, e_2, e_3]$ to $[u_1, u_2, u_3]$.
(b) Find the coordinates of the vector $(3, 2, 5)^T$ with respect to $[u_1, u_2, u_3]$.

4. Compute the Gram-Schmidt QR factorization of the matrix (15%)

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 0 & 1 \\ 2 & -4 & 2 \\ 4 & 0 & 0 \end{bmatrix}$$

5. Sketch the following conic section, giving axes of symmetry and asymptotes (if any). (15%)

$$16x_1^2 + 24x_1x_2 + 9x_2^2 - 3x_1 + 4x_2 = 5$$

6. Let $g_1(t) = t - 1$ and $g_2(t) = t^2 + t$. Using the inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)dt$ on $P_2 \subset C[0,1]$, find the orthogonal complement of Span(g_1,g_2). (20%)

試題隨卷繳回