國立臺灣大學100學年度碩士班招生考試試題

1. Express d^2y/dx^2 in terms of x and y for $4\tan y = x^3$. (10%)
2. Sketch the graph of $f(x)=\frac{1+\sqrt{x}}{1-\sqrt{x}}$, and indicate the extrema, inflection points, concavity, and asymptotes (if any). (20%)
3. Evaluate the definite integral $\int_0^8 \frac{dx}{1+\sqrt[3]{x}}$. (10%)
4. Find the area and the length of the cardioid $r=1-\cos\theta$. (10%)
5. Maximize $x^2 + y^2$ on the curve $x^4+7x^2y^2 + y^4=1$. (10%)
6. Determine whether the series $\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$ converges or diverges. (10%)
7. Find the Taylor polynomial $P_5(x)$, its remainder, and the interval of convergence for the given function $f(x) = e^x \sin x$. (20%)
8. Evaluate the integral $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_0^{\sqrt{9-x^2-y^2}} \frac{1}{\sqrt{x^2+y^2}} dzdxdy$. (10%)

試題隨卷繳回