1. (25%)

An object of mass m is thrown vertically upward from the earth's surface with initial velocity $\dot{z}(0) = v_0$. Assume that there is a force proportional to the velocity resisting the motion (Fig. 1), and that the ratio c/m is relatively large, please derive an expression for the velocity of the object when it hits the ground upon its return.

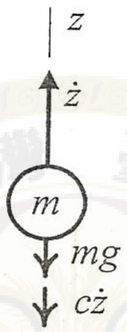

Fig.1

2. (25%)

The dynamic model of a simple harmonic oscillator can be expressed as $\ddot{\theta} + \omega^2\theta = 0$, where ω is a constant. Assuming that the equation has the general solution in time t as $\theta(t) = A\cos(\omega t) + B\sin(\omega t)$, where A and B are constants determined by initial conditions. Show that the 'phase portrait' of the system as shown in Fig. 2 has elliptical trajectory for each pair of initial conditions (A, B).

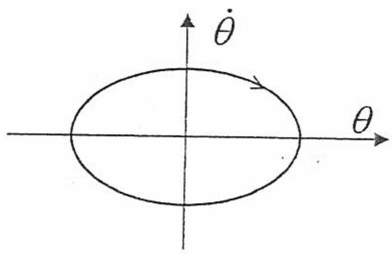

Fig. 2

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3. (25%)

Consider the system of pulleys, masses and string shown in Fig. 3. Assume that the pulleys are small, so we can neglect the pulley radius. Let the potential energy being zero along the line AB. The system is in equilibrium when its potential energy is a constant. Derive the conditions imposed on the masses so that the system remains in equilibrium.

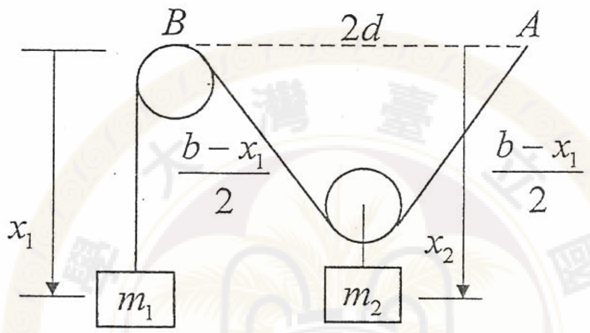

Fig. 3

4. (25%)

Figure 4 shows that atoms interact by elastic forces between each other and oscillate about their equilibrium positions. Springs between the atoms represent the elastic forces. Each of the oscillator springs have a force constant k; the force constant of the coupling spring is k_{12}. The system has two degrees of freedom, represented by the coordinates x_1 and x_2. Each coordinate is measured from the position of equilibrium. Assume the motions are oscillatory, $x_1(t) = B_1 e^{i\omega t}$; $x_2(t) = B_2 e^{i\omega t}$. The ω is the oscillation frequency, and B_1, B_2 are complex amplitudes. Please derive the oscillation frequencies of the system.

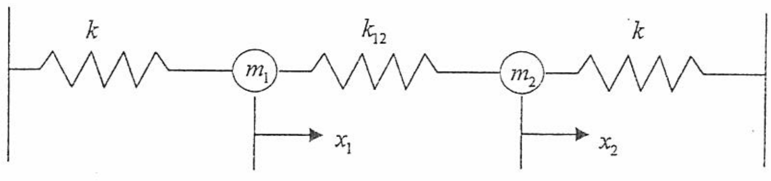

Fig. 4

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