1. (20%) Given the Bessel's equation $x^2y''+xy'+(x^2-\nu^2)y=0$ and the solution can be expressed as $y=A\,J_\nu(x)+BY_\nu(x)$.
   Find the general solution (in terms of the Bessel function) of the equation $my''+k\,e^{-\alpha t}\,y=0$ with the indicated substitution $x=\frac{2}{\alpha}\sqrt{\frac{k}{m}}\,e^{-\alpha t/2}$.

2. (20%) If $f(x)$ is a periodic function with period of T, prove that:
   $$\frac{1}{T}\int_{-T/2}^{T/2} f^2(x)dx \ge a_o^{\,2}+\frac{1}{2}\sum_{m=1}^{k}\left[a_m^{\,2}+b_m^{\,2}\right]$$
   Where $\omega_o \equiv \frac{2\pi}{T}$; $\quad a_o=\frac{1}{T}\int_{-T/2}^{T/2} f(x)dx$;
   $$a_n=\frac{2}{T}\int_{-T/2}^{T/2} f(x)\cos n\omega_o x\,dx;\quad b_n=\frac{2}{T}\int_{-T/2}^{T/2} f(x)\sin n\omega_o x\,dx;$$

3. (20%) Consider the 1D wave equation $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$ and the boundary conditions are specified as $\left.\frac{\partial u}{\partial x}\right|_{x=0}=0$ and $\left.\frac{\partial u}{\partial x}\right|_{x=L}=0$. If the initial conditions are given by $u(x,0)=1+2\cos(\frac{3\pi x}{L})+5\cos(\frac{7\pi x}{L})$ and $\left.\frac{\partial u}{\partial t}\right|_{t=0}=0$. Find the solution of $u(x, t)$.

4. (20 %) For an Eigen value problem $\mathbf{A}\mathbf{x}=\lambda\,\mathbf{x}$, prove that the eigenvalues are real if **A** is a Hermitian matrix.

5. (20%) Consider the following two initial value problems:
   (I) $m\ddot{y}+c\dot{y}+ky=\delta(t)$ I.C. $y(0)=\dot{y}(0)=0$, $\delta(t)$: Dirac Delta function
   (II) $m\ddot{y}+c\dot{y}+ky=r(t)$ I.C. $y(0)=\dot{y}(0)=0$
   If $h(t)$ is the solution of problem (I), show that the solution of problem (II) can be expressed by:
   $$y(t)=\int_0^t h(t-\tau)\,r(\tau)\,d\tau$$

試題隨卷繳回