1. Please find the <u>general solutions</u> of the following ordinary differential equations (O.D.E.s):

   (6%) (a) $[x\sin\theta]d\theta+[x^3-2x^2\cos\theta+\cos\theta]dx=0$

   (6%) (b) $(xD^4+D^3)y=150x^4$

2. (10%) Solving the following O.D.E. by Laplace transform.

   $y''+3y'+2y=r(t)$, $r(t)=1$ if $1<t<2$ and 0 otherwise

   $y(0)=y'(0)=0$

3. (6%) (a) Find the inverse Laplace transform of F(s):

   $$F(s)=\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$$

   (6%) (b) Find the Laplace transform of $f(t)$:

   $$f(t)=e^{at}\frac{\sinh t}{t}\quad (a\text{ is a constant})$$

4. For the following O.D.E.,

   $x(x-1)y''+(3x-1)y'+y=0$

   (5%) (a) Is $x=0$ an ordinary point, regular singular point, or irregular singular point?

   (10%) (b) Find power series solutions near $x=0$.

5. (10%) Prove that

   Let $\underline{A}$ be a real, symmetric matrix, then there is a real, orthogonal matrix that diagonalizes $\underline{A}$.

6. If a vector field $\underline{F}=3x^2\underline{i}+2yz\underline{j}+y^2\underline{k}$, C: the path connecting (0,1,2) and (1,-1,7).

   (6%) (a) Please show that $\underline{F}$ is a conservative vector field. Since $\underline{F}$ is a conservative vector field, there exists a potential function ϕ. Please also show the relationship between $\underline{F}$ and ϕ.

   (8%) (b) Please calculate ϕ and $\int_C \underline{F}\bullet d\underline{r}$.

見背面

7. (15%) Solving the following partial differential equation (P.D.E.) by separation of variables.

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$$

I.C. $T(x,0) = f(x)$

B.C. (1) $\frac{\partial T(0,t)}{\partial x} = 0$

(2) $\frac{\partial T(l,t)}{\partial x} = -T(l,t)$

8. You are given the following matrix $\underline{A}$:

$$\underline{A} = \begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}$$

(4%) (a) Find the eigenvalues and eigenvectors of $\underline{A}$

(4%) (b) If $\underline{A}$ is similar to $\underline{D}$ (a diagonal matrix with eigenvalues as the diagonal components), what are the transition matrix $\underline{P}$ and its inverse $\underline{P}^{-1}$?

(4%) (c) Use the results from (a) and (b) to solve the following system of O.D.E.s:

$$y_1' = 4y_2 + 9t$$
$$y_2' = -4y_1 + 5$$