*Note: 請將題號及答案標示清楚

1. (30%) Figure 1 shows two pendulums suspended from frictionless pivots and connected at their mid-points by a spring with spring constant k and natural length l. Assume that each pendulum can be represented by a mass m at the end of a massless bar of length l. Also assume that the vertical displacement of the spring can be ignored due to the small movement of the pendulums (i.e., the spring moves horizontally). The external torque is represented by $\tau(t)$.
   (a) (5%) Obtain the equations of motion.
   (b) (5%) Derive the linear approximations of $cos\theta$ and $sin\theta$ at the operation point $\theta=\theta_1=\theta_2=0$, and use them to derive the linearized equations of motion.
   Use the linearized model for the following questions.
   (c) (5%) Sketch a block diagram for the equations of motion.
   (d) (5%) Determine the transfer function $G(s) = \theta_1(s)/\tau(s)$.
   (e) (5%) Derive the poles and zeros of the system and sketch them on the s-plane.
   (f) (5%) Find the sensitivity of the transfer function $G(s)$ to the parameter variation of l.

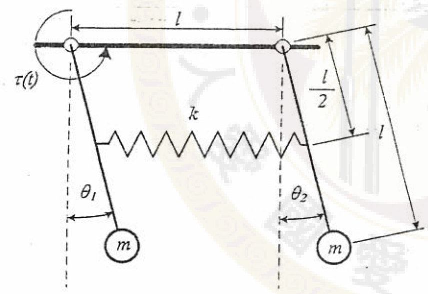

Figure 1

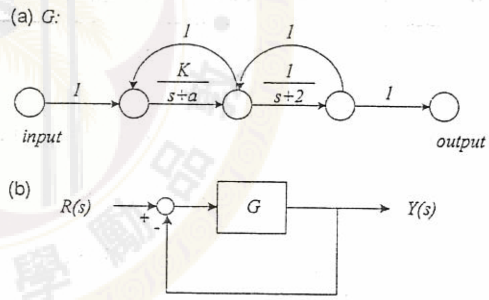

Figure 2

2. (20%) A system is shown in Figure 2.
   (a) (5%) Determine the transfer function $G(s)$ of the plant shown in Figure 2(a) by Mason's formula.
   (b) (5%) At what ranges of a and K is the plant stable?
   (c) (5%) If the input is unit step and $K>0$, is it possible to design the plant free from oscillating transient by selecting a? If yes, find the range of a. If no, explain why.
   (d) (5%) Determine the steady state error of the closed-loop system with negative unity feedback to the unit step input $r(t)$ shown in Figure 2(b).

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3. (15%) Consider Figure 3 with $L(s)=\dfrac{s+2}{s^2(s-50)}$,

   (a) (5%) Please sketch the Bode plots of $L(s)$. Please specify important slopes, frequencies, and degrees.

   (b) (5%) Please sketch the Nyquist plot of $L(s)$.

   (c) (5%) ***Using Nyquist Criterion***, please determine whether the closed-loop system is stable. If it is unstable, how many right-hand-plane poles it has?

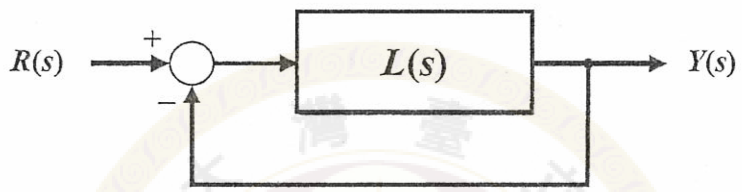

Figure 3

4. (20%) Consider Figure 4 with $G(s)=\dfrac{s-1}{s(s+100)}$,

   (a) (5%) Please sketch the root loci of the closed-loop system for a proportional control $C(s)=K$ as $K:0\to\infty$.

   (b) (5%) Using root-loci techniques, which of the following controllers would you select to stabilize the system: (1) $C(s)=\dfrac{K(s+2)}{s-2}$; (2) $C(s)=\dfrac{K(s-2)}{s+2}$; (3) $C(s)=\dfrac{K(s+4)}{s+2}$; where $K:0\to\infty$. Please explain your choice by drawing the root loci of the closed-loop system.

   (c) (5%) Using the controller selected in (b), decide the range of K such that the closed-loop system is stable.

   (d) (5%) Please sketch the Bode plot of $C(s)G(s)$ when K=1. Please specify important slopes, frequencies, and degrees.

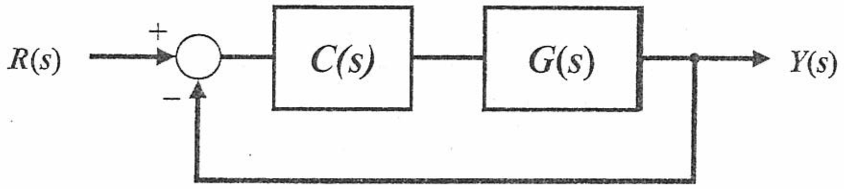

Figure 4

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5. (15%) Consider the closed-loop system of Figure 4, where the Nichol's Plot of $G(s)$ is given in Figure 5
   (a) (5%) Find the Gain Margin α of the system.
   (b) (5%) Find the Phase Margin ϕ of the system.
   (c) (5%) Suppose $C(s)=\alpha$ found in (a), decide the location of two poles of the closed-loop system.

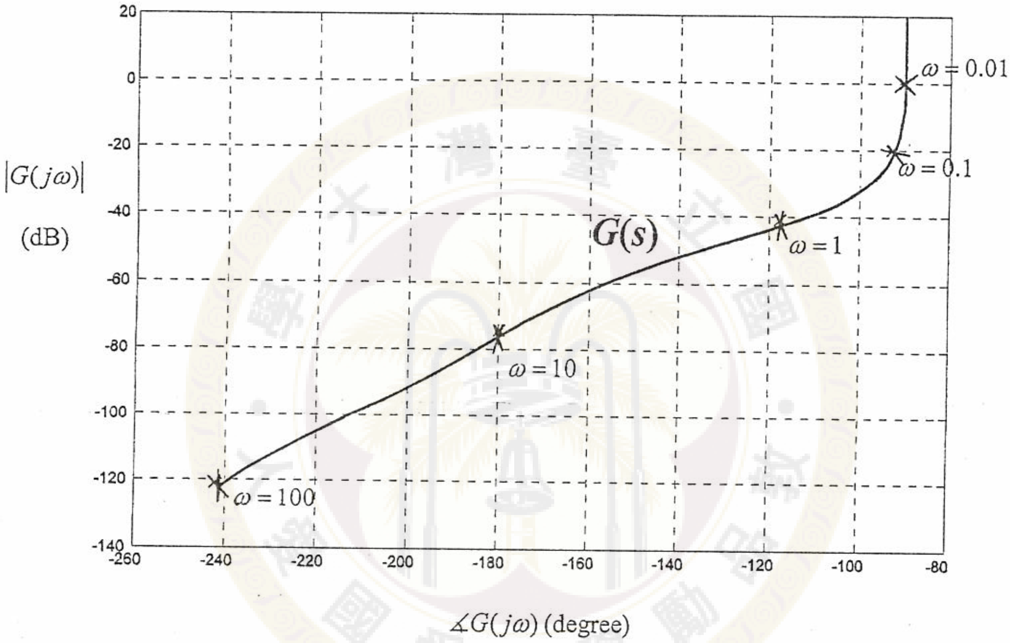

Figure 5