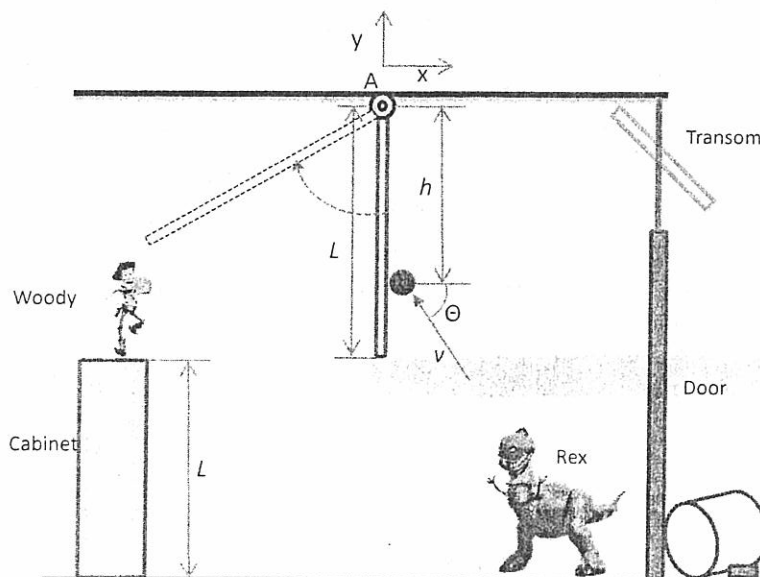




3. Andy's loyal toys find themselves in a daycare where untamed tots with their sticky little fingers do not play nice. So, it's all for one and one for all to plan their great escape from the daycare. Woody plans to escape through the transom above the door, which has been locked by the ill-willed teddy bear called Lots-o'-Huggin' Bear, but the transom is too high for him to reach directly. Thus, he tries to take advantage of the bar hanging from the ceiling to get to the top of the door.

As shown in the figure below, the bar, whose mass is  $m_1$ , whose length is  $L$  and whose moment of inertial about its pivot is  $I_A$ , is at rest in the vertical position. He asks Rex to obliquely strike the bar with a ball, whose mass is  $m_2$ , so that he can reach the bar on top of the cabinet as it swings. Assume the coefficient of restitution for the collision between the bar and the ball is  $\epsilon$  and use  $\Delta t$  to denote the duration of the impulse. Also, consider the ball to be a particle, which means that we need not consider its rotation.

- Determine the angular velocity of the bar  $\overline{\omega}_f$  and the velocity of the ball  $\overline{v}_{2f}$  immediately after the collision. (10%)
- Determine the impulse of the pivot force during the collision. (5%)
- Is there a distance  $h$  for the impact that minimizes this impulse? (5%)
- Determine the total energy loss as a result of the collision by comparing the total kinetic energy before and after the collision. How does the angle of the impact  $\Theta$  affect the energy loss? (5%)



4. Brave Woody successfully climbs out the transom and jumps on a heavy barrel, which cannot freely roll on the ground due to the bricks placed underneath it. As shown in the figure blow, Woody is modeled as a homogeneous box, whose mass is  $m$  and whose centroid  $G$  is above the contact  $C$ .

- (a) Assuming that the box does not slip, derive the differential equation of motion governing the angle  $\phi$  by which the box rotates away from horizontal. (10%)
- (b) The governing equation you just derive in (a) is nonlinear, and an analytical solution would be difficult. Thus, it normally resorts to numerical techniques to obtain the solution. Here, we want to derive test solutions to verify the numerical analysis by considering  $\phi$  to be small. Use the approximations  $\cos\phi \approx 1$ ,  $\sin\phi \approx \phi$ , and drop any terms that have quadratic or higher powers of  $\phi$  to linearize the governing equation. (5%)
- (c) If  $h > 2R$ , is the response of the linearized equation obtained in (b) a good approximation for the nonlinear governing equation derived in (a)? Is there any limitation? Explain. (Hint: Consider the solution form of the linearized equation.) (5%)
- (d) If  $h < 2R$ , what is the influence of the length  $b$  on the response of  $\phi$ ? (5%)

