1.(25%) A vertical straight column with rigidity EI is subjected to a compressive load P. When the load P is over the critical loading the column will buckle under the slightest disturbance, and the buckling equation and boundary conditions are given in the following equations:

$$v''(x) + k^2 v(x) = k^2\delta + \frac{M_0}{EI}, \quad 0 < x < L,$$
$$v(0) = 0, \quad v'(0) = 0, \quad v(L) = \delta, \quad v'(L) = 0,$$

where M_0 and δ are, respectively, the unknown moment and deflection, and $k^2 = P/(EI)$.

(a)(5%) By observing the boundary conditions at $x = 0$, what type of support is it at the bottom (fixed, pinned, or free)?

(b)(10%) Determine the first two critical loadings P_c.

(c)(10%) Determine the first two buckled modes of $v(x) \neq 0$.

2.(25%) A plane stress is given by

$$\tau = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}.$$

The stress vector $\mathbf{T}$ acts on an inclined plane whose normal $\mathbf{n}$ making an angle θ with respect to the x-axis. Correspondingly, $\mathbf{s}$ denotes the shear direction along the plane with $\mathbf{s} \cdot \mathbf{n} = 0$. The Cauchy formula in terms of τ, $\mathbf{T}$, and $\mathbf{n}$ is $\tau\mathbf{n} = \mathbf{T}$.

(a)(10%) By using $N = \mathbf{T} \cdot \mathbf{n}$ and $S = \mathbf{T} \cdot \mathbf{s}$, please derive the normal stress N and the shear stress S in terms of σ_x, σ_y, τ_{xy} and θ (stress transformation formulas).

(b)(5%) Can you find an angle θ_p and write out θ_p such that N is a maximum? In this orientation what is the value of S?

(c)(5%) Can you find an angle θ_s and write out θ_s such that S is a maximum? In this orientation what is the value of N?

(d)(5%) If $\sigma_x\sigma_y - \tau_{xy}^2 > 0$, can you find an orientation such that N is zero?

3.(20%) A horizontal beam is 10 m long, 1 m wide, and 1 m depth, and made of isotropic, linearly elastic material with Young's modulus $E = 120$ GPa and Poisson's ratio $\nu = 0.2$. It is subjected to a uniformly distributed load $q = 8$ kN/m acting vertically downwards over the entire span $0 \leq x \leq 10$ m. The beam is fixed at the left end $x = 0$.

(a)(10%) If the right end $x = 10$ m of the beam is free, derive the deflection curve $w(x)$ for $0 \leq x \leq 10$ m.

(b)(10%) If the right end $x = 10$ m is supported by an elastic spring with spring stiffness $k = 15$ MN/m, determine the reaction force induced in the spring support.

4.(12%) Suppose the material under consideration is isotropic, linearly elastic.

(a)(3%) What is the formula expressing the shear modulus G in terms of Young's modulus E and Poisson's ratio ν? For the material in Problem 3, determine the value of the shear modulus G.

(b)(5%) For the beam in Problem 3(a), calculate the shear stress $\tau(x, y, z)$ and the shear strain $\gamma(x, y, z)$ on the cross section $-0.5 \leq y \leq 0.5$ m, $-0.5 \leq z \leq 0.5$ m at the fixed end $x = 0$.

(c)(4%) Check and compare the value of G you just obtained in (a) with the value of G found in $\tau = G\gamma$ where the values of τ and γ are those you just calculated in (b). Please comment on your check and comparison.

5.(18%) A horizontal member is 10 m long, 1 m wide, and 1 m depth, and made of isotropic, linearly elastic material with Young's modulus $E = 120$ GPa and the thermal expansion coefficient α. The temperature on the upper face is raised by an amount $\Delta T = 10^{-5}/\alpha$, but the temperature on the lower face remains unchanged. In between the temperature varies linearly.

(a)(9%) Suppose the member is a cantilever. Find the axial strain and curvature.

(b)(9%) Suppose the member is fixed at both ends. Find the axial force and moment induced in the member.

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