- 必須說明理由或推導過程, 否則不予計分。
- 請 依題號依序作答。

1. (10%) You run an OLS regression of *earnings* on *education* (years) and IQ (an intelligence test), and then dropped IQ and run *earnings* on *education* alone. What would happen to the coefficient on *education*? Why?

2. (10%) If you run a regression of *output* on *labor* and *capital*, and then run a regression of *output* on *labor*, *capital*, and *land*, in what situation(s) will the R^2 in the second regression equal the R^2 from the first regression? Can it ever be smaller?

3. (15%) The following wage regression was run using 528 observations:

$$\underset{}{log(wage)} = \underset{(0.146)}{1.25} + \underset{(0.011)}{0.07}\ edu - \underset{(0.218)}{0.56}\ female + \underset{(0.016)}{0.03}\ female \times edu + \widehat{u} \qquad (1)$$

where edu is years of schooling, $female$ is a female dummy variable: $female = 1$ if female and $female = 0$ if male.

(A) What is the return to schooling for females?

(B) What is the predicted percentage difference in the wages of a woman and a man having the same years of schooling?

(C) Suppose the original sample was split by gender and the two following regressions $log(wage) = \widehat{f_0} + \widehat{f_1} edu + \widehat{u}_f$ (for females) and $log(wage) = \widehat{m}_0 + \widehat{m}_1 edu + \widehat{u}_m$ (for males) run separately. Can you use equation (1) to determine the $\widehat{f}$'s and $\widehat{m}$'s?

4. (15%) The model is

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$
$$E(u_i|X_i) = 0, \qquad E(u_i^2|X_i) = \sigma^2.$$

Suppose that you know that the true value of β_1 is 1.

(A) Show that in this case it makes sense to estimate β_0 by

$$\widetilde{\beta}_0 = \overline{Y} - \overline{X}.$$

where $\overline{Y}$ and $\overline{X}$ are sample averages of Y_i and X_i.

(B) Find $E(\widetilde{\beta}_0)$.

(C) Find $Var(\widetilde{\beta}_0)$.

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5. (15%) Suppose X and Y are discrete random variables with the following probabilities:

$$\begin{array}{ll} \Pr(X=0,Y=0)=0.05 & \Pr(X=0,Y=1)=0.1 \\ \Pr(X=2,Y=0)=0.2 & \Pr(X=2,Y=1)=0.15 \\ \Pr(X=4,Y=0)=0.15 & \Pr(X=4,Y=1)=0.2 \\ \Pr(X=6,Y=0)=0 & \Pr(X=6,Y=1)=0.15. \end{array}$$

Answer the following questions.

(A) What is the variance of X?

(B) What is the variance of X conditional on $Y=0$?

(C) Suppose $\{(X_1,Y_1),(X_2,Y_2),\dots(X_n,Y_n)\}$ are drawn independently from the same distribution as (X,Y). Define a new random variable W_n as the average of $\{Y_1,Y_2,\dots,Y_n\}$:

$$W_n \equiv \frac{\sum_{i=1}^{n} Y_i}{n}.$$

What is the limit of the median in the distribution of W_n when n approaches infinity ($\lim_{n\to\infty} Median(W_n)$)?

6. (15%) Suppose X is a continuous random variable with the following density function:

$$f(x)=\begin{cases} (\theta+1)(1-x)^{\theta}, & \text{if } 0\le x\le 1; \\ 0, & \text{otherwise.} \end{cases},$$

where $\theta>0$ is a parameter. Answer the following questions.

(A) What is the mean of X?

(B) Let $Y=X^2$. What is the density function of Y?

(C) Suppose that $(x_1,x_2,\dots,x_n)$ is a random sample drawn from the distribution of X. What is the maximum likelihood estimator of θ?

7. (20%) Suppose that X_1, X_2, and Y are independent random variables. Assume that $X_1\sim N(\mu_1,\sigma_1^2)$, $X_2\sim N(\mu_2,\sigma_2^2)$, $Y\sim Bernoulli(0.5)$ (i.e. $\Pr(Y=1)=0.5$ and $\Pr(Y=0)=0.5$). Define a new random variable:

$$Z=\begin{cases} X_1, & \text{if } Y=0; \\ X_2, & \text{if } Y=1. \end{cases}$$

Answer the following questions.

(A) What is the mean of Z?

(B) What is the variance of Z?

(C) Is the random variable Z distributed as a normal distribution?

(D) What is the covariance between X_1 and Z?

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