1. (15%) Which of the following is (are) correct? (複選題)
   (a)   Tr(AB)=Tr(BA) holds for arbitrary finite or infinite dimensional matrices A, B.
   (b)   For all 3x3 matrices A, it is possible to express det(A) as a function of Tr(A), $Tr(A^2)$, $Tr(A^3)$.
   (c)   The 3 functions $1, x, x^2$ are linearly independent.
   (d)   The matrix U=exp(A) is unitary whenever A is anti-Hermitian.
   (e)   Arbitrary vectors A, B, C in 3 dimensional Euclidean space satisfy the relation $A \times (B \times C) = B(A \cdot C) + C(B \cdot A)$, where " $\times$ " denotes the cross product, and " $\cdot$ " denotes the inner product.

2. (a)(15%) Find the eigenvalues and corresponding eigenvectors of the matrix
$$M = \begin{pmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{pmatrix}.$$
   (b)(10%) Find the value of $Tr(f(M))$ for $f(x) = x^2 + \sin(x)$.

3. (15%) Find the general solution for $y(x)$ to the differential equation
$$\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + y = \exp(x).$$

4. (15%) Solve the differential equation
$$\frac{dy}{dx} = \frac{y^2}{1+x^2}$$
for the solution with $y(0) = 1$.

5. (15%) For all non-negative integers n, evaluate the integral
$$\int_{-\infty}^{\infty} x^n \exp(-x^2/2)\, dx.$$

6. (15%) Define $F(k)$ by the equality
$$\int_{-\infty}^{\infty} F(k)\exp(ikx)\, dk = \frac{1}{1+x^2}.$$
Find $F(k)$.

試題隨卷繳回