1. (10%) Let $X$ be a continuous random variable with the distribution function $F(x)$. Derive the corresponding distribution function of $F(X)$.

2. (10%) Let $X$ and $Y$ be mutually independent Chi-square random variables with the degrees of freedom $m$ and $n$. Compute the mean of $U = (nX)/(mY)$ for $n > 2$.

3. (10%) Let $X_1, \cdots, X_n$ be a random sample from a population with density function $f(x) = 0.5\sigma^{-1}\exp(-|x|/\sigma)I_{(-\infty,\infty)}(x)$. Find a moment estimator of $\sigma$.

4 Let $X_1, \cdots, X_n$ be a random sample from an exponential distribution with the density function $f(x) = \tau^{-1}\exp(-x/\tau)I_{(0,\infty)}(x)$.

(4a) (10%) Derive the sampling distribution of the maximum likelihood estimator of $\tau$.

(4b) (10%) Find the uniformly minimum variance unbiased estimator of $\tau$.

5. Let $X_1, \cdots, X_n$ be a random sample from a geometric distribution $P(X = x) = \theta(1-\theta)^{x-1}I_{\{1,2,\cdots\}}(x)$, and $\theta$ have a uniform prior distribution on $[0, 1]$.

(5a) (10%) Derive the posterior distribution of $\theta$.

(5b) (10%) Based on the loss function $L(\theta, \delta(X_1, \cdots, X_n)) = (\delta(X_1, \cdots, X_n) - \theta)^2$, find the Bayes estimator of $\theta$.

6. Suppose that $X_1, \cdots, X_m$ are independent with $X_i \sim Binomial(n, p_i)$.

(6a) (8%) Derive a likelihood ratio test for the null hypothesis $H_0 : p_1 = \cdots = p_m$ versus the alternative hypothesis $H_A : p_i \neq p_j$ for some $i \neq j$.

(6b) (7%) What is the large sample distribution of the test statistic?

7. (15%) Define the following terms:

(7a) random variable. (7b) convergence in distribution. (7c) strong law of large numbers.

試題隨卷繳回