Problem 1. (20%)
(a)(7%) Explain how the second degree Lagrange polynomial with three distinct nodes $x_0 = a$, $x_1 = a + h$, and $x_2 = a + 2h = b$, where $h = \frac{b-a}{2} > 0$, can be used to approximate a one dimensional smooth real function $f(x)$.
(b)(7%) Expand the function $f(x)$ in the third degree Taylor polynomial about x_1. Remember to include the error term involving the fourth derivative of $f(x)$.
(c)(6%) Compare the two approaches in (a) and (b) to approximate the function.

Problem 2. (25%)
(a)(10%) Derive the Simpson's rule to approximate $\int_a^b f(x)dx$ from the third degree Taylor polynomial for a smooth function $f(x)$.
(b)(8%) Explain the idea of the composite Simpson's rule to approximate the the definite integral as detail as you can.
(c)(7%) Consider using the composite Simpson's rule to approximate $\int_0^\pi \sin x dx$. Find a number of uniform sub-intervals n for $[0, \pi]$ leading to an absolute error less than 10^{-5}. Note that the error of the composite Simpson's rule is $-\frac{b-a}{180}h^4 f^{(4)}(\mu)$ under suitable assumptions.

Problem 3. (20%)
(a)(8%) Derive the Euler method for solving a initial value problem $y'(t) = f(t, y(t))$, with $y(t_0) = y_0$. Also explain the Euler method geometrically.
(b)(4%) What are the motivations for developing multistep methods like Adams-Bashforth Three-Step Explicit Method and Adams-Moulton Three-Step Implicit Method?
(c)(4%) Briefly describe the idea of a predictor-corrector method to solve an initial value problem for ordinary differential equations.
(d)(4%) Briefly illustrate another idea, other than the ones mentioned in (b) and (c), for improving the Euler method.

Problem 4. (20%)
Consider the following MATLAB function.

```
01   function S = XYZ(A)
02   if (A==0)
03      [  ?  ]
04   elseif (A<0)
05      [  ?  ]
06   else
07      L = A; W = 1;
08      while (abs(L-W)>(10^-15)*L)
09         L = (L+W)/2;
10         W = A/L;
11      end
12      S=L;
13   end
```

(a)(6%) What is the purpose of the MATLAB function?
(b)(6%) Is the stopping criterion in line 08 a reasonable choice? What are your reasons?
(c)(4%) Fill in the blank in line 03 to achieve the goal of the function.
(d)(4%) Fill in the blank in line 05 to achieve the goal of the function.

Problem 5. (15%)
What is the most interesting scientific computation problem you have ever solved? Try to describe the following items as detail as possible: the problem, the difficulties/challenges, the algorithms/methods/ideas, the computer codes, the results/findings, and possible future directions.

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