(1) (11 %) Evaluate $\lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$

(2) (12 %) (a) Evaluate the limit $f(x)=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{x(1-x)}{k+(n-k)x}$, $x\in[0,1]$.
(b) Find the extreme values of $f(x)$ on $[0,1]$.

(3) (11 %) Consider the curve $C: r=\sin\theta$. Let $P=(r,\theta)\in C, r\neq 0,1$. The tangent line of C at P, the x-axis and $\overline{OP}$ form a triangle where O is the origin. Find the area of the triangle.

(4) (11 %) Show that $(1-4x)^{-\frac{1}{2}}=\sum_{n=0}^{\infty}\binom{2n}{n}x^n, |x|<\frac{1}{4}$.

(5) (11 %) Let $f(x,y)=\begin{cases} x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y}, & xy\neq 0 \\ 0, & xy=0. \end{cases}$ Find $\frac{\partial^2 f}{\partial x\partial y}(0,0)$.

(6) (11 %) The plane $4x-3y-z=5$ intersects the cone $x^2+y^2=z^2$ in an ellipse. Find the highest and the lowest points on the ellipse.

(7) (11 %) Evaluate $\int_0^{\frac{2}{3}}\int_y^{1-\frac{y}{2}}(2x+y)e^{y-x}dxdy+\int_{-\frac{2}{3}}^{0}\int_{-2y}^{1-\frac{y}{2}}(2x+y)e^{y-x}dxdy$.

(8) (11 %) Find the volume of the solid bounded by the surface $(x^2+y^2+z^2)^2=x^2+y^2-z^2$.

(9) (11 %) Let C be the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$. Evaluate $\oint_C(x^{\frac{4}{3}}+y^{\frac{4}{3}})\,ds$

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