1 (12%)
Let X be a Gaussian random variable with mean = μ and variance = σ^2,
Find the third and fourth moments: $E[X^3]$ and $E[X^4]$.

2 (18%)
Let $x_n; n = 1 \cdots N$ be the N independent samples of a random variable X
1) What are the sample mean (m) and sample variance (v) of the sample data?
2) find the expected values $E[m]$ and $E[v]$

3 (10%)
Let the pdf's of two random variables X and Y be $f_X(x) = \alpha e^{-\alpha x}; x \geq 0$ and $f_Y(y) = \beta e^{-\beta y}; y \geq 0$,
Find the pdf of the random variable $Z = X + Y$.

4 (15%)
For a system described by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 10x(t)$$

where $x(t)$ and $y(t)$ are the input and output signals.
1 Find the transfer function of the system.
2 Find the solution of $y(t)$ for $x(t) = 5cos\,(2t)$.

5 (15%)
Solve the solution $y(t)$ for the following differential equation

$$\frac{dy}{dt} = y^2 e^{2t}$$

with $y(0) = 2$.

6 (15%)
Find the null space of the following matrix

$$\begin{bmatrix} 1 & 9 & 8 & 9 \\ 0 & 5 & 1 & 4 \\ 1 & -1 & 6 & 1 \end{bmatrix}$$

7 (15%)
Let $\lambda_n; n = 1 \cdots N$ be the eigenvalues of a $N \times N$ matrix A, show that $det(A) = \prod_{n=1}^{N} \lambda_n$.

試題隨卷繳回