第一大題 1-5 選擇題考生請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。
Questions 1-5 are mixed with **single-choice** and **multiple-choice** questions.

1. (10%) Suppose a 32-pound weight stretches a spring 2 feet. If the weight is release from rest at the equilibrium position, find the equation of motion $x(t)$ if an impressed force $f(t)=\sin(t)$ acts on the system for $0\le t<2\pi$, and is then removed. Ignore any damping forces.

(A) $f(t)=\sin(t)-\sin(t)U(t-2\pi)$, (B) $x(t)=\frac{-1}{10}\sin 4t+\frac{1}{30}\sin t, 0\le t<2\pi$,

(C) $x(t)=\frac{-1}{60}\sin 4t+\frac{1}{15}\sin t, 0\le t<2\pi$, (D) $x(t)=\frac{-1}{30}\sin 4t+\frac{1}{5}\sin t, 0\le t<2\pi$,

2. (10%) A uniform 10-foot-long chain is coiled loosely on the ground. One end of the chain is pulled vertically upward by means of constant force of 2 pounds. The chain weights 1 pound per foot. Determine the height of the end above ground level, x(t), at time t.

(A) $x\frac{d^2x}{dt^2}+(\frac{dx}{dt})^2+16x=160$, (B) $x\frac{d^2x}{dt^2}+(\frac{dx}{dt})^2+32x=64$,

(C) $x\frac{d^2x}{dt^2}+(\frac{dx}{dt})^2+32x=160$, (D) $x=3-3(1-\frac{4}{3}t)^2$, (E) $x=\frac{15}{2}-\frac{15}{2}(1-\frac{4\sqrt{10}}{15}t)^2$

3. (10%) A semi-infinite plate coincides with the region defined by $0\le x\le\pi, y\ge 0$. The right end is held at temperature $e^{-y}$, and the left end is held at temperature zero. The bottom of the plate is held at temperature $f(x)$. Find the steady-state temperature in the plate: $u(x,y)=\sum_{n=1}^{\infty}\frac{2}{\pi}A_n e^{-ny}+\frac{2}{\pi}\int_0^{\infty}B(\alpha)\sin(\alpha y)d\alpha$

(A) $A_n=\sin(nx)\int_0^{\pi}f(x)\sin(nx)dx$, (B) $A_n=\int_0^{\pi}f(x)\sin(nx)dx$,

(C) $B(\alpha)=\frac{\sinh(\alpha x)}{(1+\alpha^2)\sinh(\alpha\pi)}$, (D) $B(\alpha)=\frac{\alpha\sinh(\alpha x)}{(1+\alpha^2)\sinh(\alpha\pi)}$

4. (10%) Use the power series method to solve the DE $xy''+2y'-xy=0$, $y=c_1y_1+c_2y_2$

(A) $y_1=\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1}$, (B) $y_1=\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n}$, (C) $y_2=\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n}$,

(D) $y_2=\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n-1}$, (E) $y_1=\frac{\sinh(x)}{x}$

5. (10%) Solve the given equation $\int_0^t f(\tau)f(t-\tau)d\tau=6t^3$,

(A) $f(t)=6t$, (B) $f(t)=3\sqrt{2}t$, (C) $f(t)=\sqrt{6}t$, (D) $f(t)=-6t$, (E) $f(t)=-2t$

<u>第二大題考生應作答於『試卷』</u>

1. Toss 3 dice together (each with 6 unbiased faces: 1 to 6). Let Z be the median of the results. For example, if the results are 4, 3 and 6, then the median is 4. If the results are 4, 2, and 2, the median is 2. Please find the average of Z: E[Z] (10%)

2. X and Y are two independent random variables. The cumulative density function (C.D.F) of X is shown in Figure 1 and the probability density function (P.D.F) of Y is shown in Figure 2. Please (1) find the value of a+b+c (4%) (2) find the P.D.F of Z=X+Y (6%)

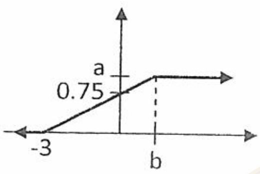


Figure 1: C.D.F. of X

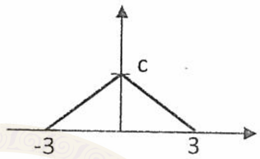


Figure 2: P.D.F. of Y

3. Let X and Y are two discrete random variables. The sample space of X is $\{x_1, x_2, \ldots, x_p\}$ and the sample space of Y is $\{y_1, y_2, \ldots, y_q\}$. The mean values of X and Y are denoted as $E[X]=M_x$ and $E[y]=M_y$, respectively. Please show that (1) if X and Y are independent of each other, then $E[XY]= M_x \cdot M_y$ (4%), (2) if $E[XY]= M_x \cdot M_y$, X and Y are NOT necessary independent of each other (6%)

4. Denote an arbitrary event $E_i \in S$, where S is the sample space of an experiment. If the number $P[E_i]$ is called as the probability of $E_i$, which three requirements should the function $P[\cdot]$ satisfy? 6% (Please specify each requirement in a mathematical form using only the notations given in this question)

5. Mr. Yang commutes between his home and office by foot every day. He has **ONLY** one umbrella, either placed in his home or the office. As long as the umbrella is with him and it is raining outside, he takes his umbrella when going out. If it is not raining, Mr. Yang will not take the umbrella even if his umbrella is nearby. In Taipei, it rains with a probability of 1/3 in the morning rush hour, and with a probability of 1/2 in the evening rush hour (independently). Assume that Mr. Yang has its umbrella with him at home in the morning of Day 1. Let X be the number of commutes, counted starting from Day 1, **BEFORE** Mr. Yang gets wet. That is, X=0 represents the event that Mr. Yang gets wet in the morning of Day 1, and X=1 represents the event that Mr. Yang gets wet in the evening of Day 1. (Note that Mr. Yang commutes twice per day, one from home to school in the morning, and the other from school back to home at night). Please calculate (1) Prob[Mr. Yang gets wet] 3%, (2) E[X]. (8%) (3) Is it better to have the umbrella at home in the morning of Day 1? Please justify your answer (3%)

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