

- (10%) Consider a continuous-time input signal $x(t) = Au(t)$ and an LTI system with unit impulse response $h(t) = Be^{-at}u(t)$.
 - (5%) Find and sketch the output signal $y(t) = x(t) * h(t)$. In particular, denote the steady-state value that $y(t)$ approaches on the sketch.
 - (5%) Is it possible to find a deconvolving function $g(t)$ such that $z(t) = y(t) * g(t) = x(t)$? If so, find it. If not, explain why not.
- (10%) Consider a discrete-time LTI system with frequency response $H(e^{j\omega})$ as shown in the following:

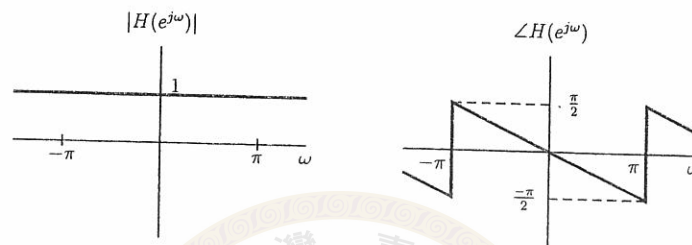


Figure 1: Magnitude and phase of the frequency response $H(e^{j\omega})$

If the input sequence $x[n]$ to the LTI system is

$$x[n] = \cos\left(\frac{5\pi}{2}n - \frac{\pi}{4}\right),$$

find and sketch the output sequence $y[n]$ of the LTI system.

- (10%) Consider the continuous-time signal

$$x(t) = \sum_{k=0}^{\infty} 2^{-k/2} \cos(40\pi kt).$$

To design a lowpass filter that would remove no more than 5% of the signal energy, what should the cutoff frequency of the filter be?

- (10%) Consider a stable, discrete-time LTI system with the input-output relationship as shown in the following:

$$y[n-1] - \frac{10}{3}y[n] + y[n+1] = x[n],$$

where $x[n]$ and $y[n]$ are the input and output sequences respectively. Find the unit impulse response $h[n]$ of the system.

- (10%) A discrete-time signal $x[n]$ has the following properties:

- $x[n]$ is real and odd.
- $x[n]$ is periodic with period $N = 6$.
- $\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = 10$.
- $\sum_{n=\langle N \rangle} (-1)^{n/3} x[n] = 6j$.
- $x[1] > 0$.

Find an expression of $x[n]$ in the form of sines and cosines.

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6. Answer "True" or "False" to each of the following statements.
- (a) (2%) For BPSK demodulation in AWGN, if a-priori probabilities are equal, then the maximum likelihood (ML) decision rule is equivalent to the maximum a-posteriori probability (MAP) decision rule.
 - (b) (2%) For BPSK demodulation, if the channel is not AWGN, then the ML decision rule may not be equivalent to the MAP decision rule even if a-priori probabilities are equal.
 - (c) (2%) For BPSK demodulation in AWGN, if a-priori probabilities are equal, then the minimum distance (MD) decision rule is equivalent to the MAP decision rule.
 - (d) (2%) For BPSK demodulation, if the channel is not AWGN, then the MD decision rule may not be equivalent to the MAP decision rule even if a-priori probabilities are equal.
 - (e) (2%) For BPSK demodulation in AWGN, if a-priori probabilities are not equal, then the MD decision rule may not be equivalent to the ML decision rule.
7. Consider a simple channel code which consists of 2 possible codewords: $\mathcal{C} = \{0000, 1101\}$. A codeword $\mathbf{c} = c_1c_2c_3c_4$ is randomly chosen from \mathcal{C} . The modulation used is 4-PAM with the constellation shown in Figure 2. The coded bits c_1c_2 are modulated into symbol s_1 ; c_3c_4 modulated into s_2 . Symbols s_1, s_2 are transmitted separately in AWGN. The receiver receives $r_1 = s_1 + n_1$, and $r_2 = s_2 + n_2$, respectively. The noises n_1 and n_2 are zero mean with variance σ^2 .

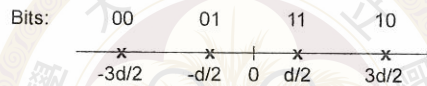


Figure 2: 4-PAM signal constellation in Problem 7

- (a) (5%) We first demodulate r_1, r_2 respectively to obtain bits $\hat{c}_1\hat{c}_2\hat{c}_3\hat{c}_4$. Suppose $\mathbf{c} = 0000$ is chosen and sent. What is the probability of seeing $\hat{c}_1\hat{c}_2\hat{c}_3\hat{c}_4 = 0101$? Express your answer in the most simplified form using d, σ , and $Q(\cdot)$.
 - (b) (5%) If the optimal **HARD** decision decoder is then used for decoding. Among all 16 possible realizations of $\hat{c}_1\hat{c}_2\hat{c}_3\hat{c}_4$, which would lead to the **HARD** decision decoding result $\hat{\mathbf{c}} = 0000$? (Please list them following the order of binary representations.)
 - (c) (5%) Compute the decoding error probability of the optimal **HARD** decision decoding if codeword $\mathbf{c} = 0000$ is chosen and sent. Express your answer in the most simplified form using d, σ , and $Q(\cdot)$. (Note: this subproblem is more time-consuming.)
 - (d) (5%) Consider the case when the optimal **SOFT** decision decoding is used. Derive the decision rule for the optimal **SOFT** decision decoder to determine which codeword is sent based on the received signals r_1 and r_2 . Express the decision rule in the most simplified form.
 - (e) (5%) Again consider the case when the optimal **SOFT** decision decoding is used. Compute the decoding error probability if codeword $\mathbf{c} = 0000$ is chosen and sent. Express your answer in the most simplified form using d, σ , and $Q(\cdot)$.
8. (5%) Consider a convolutional code encoder with generators $g_1 = [1011]$ and $g_2 = [1101]$. Plot the encoder structure of this convolutional code. What is the output of the first branch when we feed the encoder with the input sequence 1010?
9. Consider a communication system operating with the signal constellation shown in Figure 3. Both signal points are equally likely to be chosen for transmission. The noise at the receiver is very small and thus can be ignored. However, there exists an interference $\mathbf{z} = (z_x, z_y)$ in the channel where z_x and z_y are independently exponentially distributed with pdf

$$f(z_x) = e^{-z_x} u(z_x), \quad f(z_y) = e^{-z_y} u(z_y).$$

Note that $u(\cdot)$ is the unit step function, $u(t) = 1$ if $t \geq 0$; and 0 otherwise. If signal \mathbf{s}_i is sent ($i = 0$ or 1), the receiver receives signal $\mathbf{r} = \mathbf{s}_i + \mathbf{z} = (r_x, r_y)$ where $r_x = s_{ix} + z_x$ and $r_y = s_{iy} + z_y$.

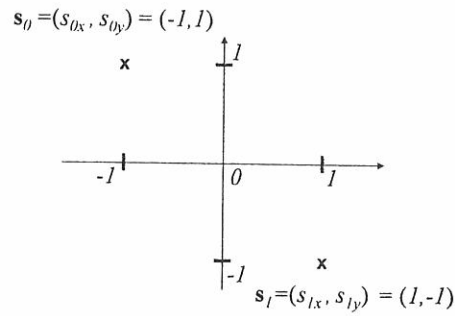


Figure 3: Signal constellation in Problem 9.

- (a) (5%) Derive the MAP decision rule and express it in the most simplified form. Plot the decision region for each symbol according to the MAP decision rule. You should label your figure clearly and correctly without any ambiguity.
- (b) (5%) What is the probability of error for the MAP receiver? The answer has to be simplified to its simplest form. If there is any computable integral in your answer, it should be computed. Do NOT leave computable integrals in integral form in your final answer.

