1. [20%] Soft-drink cans are filed by an automated filling machine. The mean fill volume is 12.1 fluid ounces, and the standard deviation is 0.05 fluid ounce. Assume that the fill volumes of the cans are independent, normal random variables. What is the probability that the average volume of 10 cans selected from this process is less than 12 fluid ounces?

2. [20%] An article in Concrete Research presented data on compressive strength x and intrinsic permeability y of various concrete mixes and cures. Summary quantities are $n = 14$, $\Sigma y_i = 572$, $\Sigma y_i^2 = 23{,}530$, $\Sigma x_i = 43$, $\Sigma x_i^2 = 157.42$, and $\Sigma x_i y_i = 1697.80$. Assume that the two variables are related according to the simple linear regression model.
   (a) Calculate the least squares estimates of the slope and intercept.
   (b) Use the equation of the fitted line to predict what mean permeability would be observed when the compressive strength is $x = 4.3$.
   (c) Given a point estimate of the mean permeability when compressive strength is $x = 3.7$.

3. [20%] The elapse time X between the occurrences of two consecutive (independent) storm events (commonly known as the *inter-arrival-time*) can be characterized by a negative exponential distribution with the following probability density function:

$$f_X(x;\lambda) = \lambda e^{-\lambda x},\ \ \lambda > 0.$$

   Suppose that inter-arrival-time (in hours) of a series of storm events are observed and listed below.

   46 21 30 59 15 28 17 49 26 24

   (a) What is the maximum likelihood estimator of λ using the above observations?
   (b) If a storm event has just occurred, what is the probability that next storm event will **not** occur within the next 24 hours?

4. [20%] A random sample of size n=100 is taken from a population of unknown mean μ wherein the standard deviation is assumed to be σ=5 grams. The computed value of sample mean is $\bar{x} = 25.8$ grams.
   (a) Find the 95% confidence interval for μ.
   (b) Conduct the hypotheses test $H_0 : \mu \le 25,\ H_1 : \mu > 25$ at level of significance $\alpha = 0.05$.

5. [20%] Two random variables X and Y have the following properties: $E(X) = 10$, $E(Y) = 8, Var(X) = 16, Var(Y) = 9$, and correlation between X and Y $= \rho_{XY} = 0.75$. Calculate the variance of $3X - 2Y$.

Table of cumulative probability for standard normal distribution ($P(Z \le z) = p$)

z	1.05	1.15	1.25	1.35	1.45
p	0.8531	0.8749	0.8944	0.9115	0.9265
z	1.55	1.65	1.75	1.85	1.95
p	0.9394	0.9505	0.9599	0.9678	0.9744
z	2.05	2.15	2.25	2.35	2.45
p	0.9798	0.9842	0.9878	0.9906	0.9929
z	2.55	2.65	2.75	2.85	2.95
p	0.9946	0.9960	0.9970	0.9978	0.9984

試題隨卷繳回