(註：可使用工程型計算機)

1. Let X_i, i=1,2,3,…,n, be independent random variables each having the standard normal density of $N(0,1)=\frac{1}{\sqrt{2\pi}}\exp[-\frac{1}{2}x_i^2], -\infty < x_i < \infty$. Find the probability density function of $Z_1=\sum_{i=1}^{n} x_i$ and $Z_2=\sum_{i=1}^{n} x_i^2$. (本題同意直接引述相關定理導出，正確給 20 分)

2. Suppose the times of successive failures of a machine form a Poisson process on $[0,\infty)$ with parameter λ. What is the probability of at least one failure during the time period $(t, t+h)$, $h>0$? (正確給 10 分)

3. It is known that a sample of 12, 11.2, 13.5, 12.3, 13.8, 11.9 comes from a population with the probability density function

$$f(x;\theta)=\begin{cases}\dfrac{\theta}{x^{\theta+1}}, & x>1\\ 0, & \text{otherwise}\end{cases}$$

where $\theta>0$. Find the maximum likelihood estimate of θ. (正確給 10 分)

4. In a statistical estimate or test issue, (a) why do we apply the *student-t* distribution to measure the mean in a small sampling size? (b) why do we apply the *Chi-square* (χ^2) distribution rather an exact variance distribution to measure the variance? (what is the relationship between the *Chi-square* distribution and an exact variance distribution?) (c) why do we apply *F*-distribution to measure the difference of variance (for example, $\sigma_1^2-\sigma_2^2=\Delta$) from two populations? (d) why do we obtain more precise result if we can apply the Bayesian approach rather than the classical approach to analyze the issue? (正確給 30 分)

5. Interpret the following result which is obtained from the statistical analysis software of SAS: (a) Write down the regression equation; (b) There is a column listing degrees of freedom (DF), say: Model, Error and Total. Can you explain how they are figured out? (c) Is this regression acceptable? Why? (at least give three reasons or messages from the table); (d) What is the purpose that the software outputs those residuals? How to make a further analysis? Any improvement for this problem? (正確給 30 分)

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob > F
Model	3	399.45437	133.15146	30.984	0.0001
Error	9	38.67640	4.29738		
C Total	12	436.13077			

Root MSE	2.0730	R-square	0.9117
Dep Mean	29.03846		
C.V.	7.13885		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	39.157350	5.88705963	6.651	0.0001
X1	1	1.016100	0.19089520	5.323	0.0005
X2	1	-1.861649	0.26732550	-6.964	0.0001
X3	1	-0.343260	0.61705204	-0.556	0.5916

OBS	Y	PREDICTED	RESIDUAL	CLM_L	CLM_U	CLI_L	CLI_U
1	25.5	27.3514	-1.85141	24.1500	30.5528	21.6733	33.0295
2	31.2	32.2623	-1.06232	30.4875	34.0371	27.2482	37.2764
3	25.9	27.3495	-1.44955	24.2757	30.4234	21.7424	32.9567
4	38.4	38.3096	0.09042	35.4098	41.2093	32.7960	43.8232
5	18.4	15.5447	2.85527	11.9729	19.1165	9.6499	21.4396
6	26.7	26.1061	0.59193	23.7649	28.4512	20.8657	31.3504
7	26.4	28.2532	-1.85316	26.4222	30.0841	23.2189	33.2874
8	25.9	26.2219	-0.32185	24.0204	28.4233	21.0413	31.4024
9	32.0	32.0882	-0.08818	30.3174	33.8589	27.0755	37.1009
10	25.2	26.0676	-0.86764	24.5024	27.6329	21.1238	31.0115
11	39.7	37.2524	2.44764	34.2957	40.2090	31.7066	42.7981
12	35.7	32.4879	3.21208	29.1743	35.8016	26.7458	38.2300
13	26.5	28.2032	-1.70324	25.9771	30.4294	23.0121	33.3943

試題隨卷繳回