Write down your answers in order. You should include all the necessary calculation and reasoning.

(10%) 1. Suppose $f(x) = Ax^5 + Bx^4 + Cx + 101 \leq 101$ for all x and $f(1) = 1$. Determine the values of A, B and C.

(10%) 2. Suppose $f(x)$ is a differentiable function defined on $(-\infty, \infty)$, satisfying $f(x+y) = f(x) + f(y)$, for every x and y. Show that $f''(x) = 0$.

(10%) 3. Evaluate the definite integral $\int_0^\pi x \sin x dx$.

(10%) 4. Estimate the value of $\ln 1.1$ so that the error is smaller than 10^{-5}.

(10%) 5. Evaluate the volume of the solid bounded by the surface

$$2x^2 + 3y^2 + 3z^2 = 6.$$

(10%) 6. Suppose $F(x,y)$ and $G(x,y)$ are two differentiable functions defined on $\mathbb{R}^2 = \{(x,y) \mid x, y \in (-\infty, \infty)\}$ so that the gradients $\nabla F(x,y) = (\frac{\partial F}{\partial x}(x,y), \frac{\partial F}{\partial y}(x,y))$, $\nabla G(x,y) = (\frac{\partial G}{\partial x}(x,y), \frac{\partial G}{\partial y}(x,y))$ are always parallel in the sense that, for every (x,y), there is a number λ, possibly dependent of the point (x,y), satisfying

$$\nabla F(x,y) = \lambda \cdot \nabla G(x,y).$$

Is it true that F must be a constant multiple of G? Prove it or give a counter example.

(10%) 7. Suppose $f(x)$ is a continuous function defined on $[-1,1]$ so that $\int_a^b f(x)dx \geq 0$ for every $a, b \in [-1,1]$, $a \leq b$. Give a reason why we can or can not conclude that $f(x) \geq 0$, for every $x \in [-1,1]$.

(10%) 8. Solve the differential equation

$$y' = 100y - y^2, \quad y(0) = 1.$$

(10%) 9. Evaluate the integral $\int\int_D (x^2 - y^2) dxdy$, where

$$D = \{(x,y) \mid 0 \leq x + y \leq 8,\ 0 \leq x - y \leq 4\}.$$

(10%) 10. Determine the maximum of the function

$$f(x,y,z) = 3x^2 + 2y^2 + z^2$$

defined on the surface

$$\{(x,y,z) \mid 2x^2 + 27y^2 + 10z^2 = 12\}.$$