Write down your answers in order. You should include all the necessary calculation and reasoning.

(10%) 1. Let $g(y)$ be the *inverse* function of $y = f(x)$ with $g(0) = 1$. Find $g''(0)$, knowing that $f'(x) = \sqrt{x^3+1}$.

(10%) 2. Determine the values of A, B and C so that 101 occurs as the *maximal* value of the function

$$f(x) = Ax^5 - x^4 + Bx^2 + C,$$

at a *unique* point on $(-\infty, \infty)$.

(10%) 3. Evaluate the definite integral $\int_0^\pi e^x \cos x dx$.

(10%) 4. Determine the values of A,B, C, D and E, given that

$$\lim_{x\to 0} \frac{\ln(x+1) - (A + Bx + Cx^2 + Dx^3 + Ex^4)}{x^5} = 0.$$

(10%) 5. Find the volume of the solid bounded by the surface

$$x^2 + 3y^2 + z^2 = 1.$$

(10%) 6. Evaluate the integral $\int_0^1 \int_y^1 x\sqrt{x^3+1}dxdy$.

(10%) 7. Suppose $f(x)$ is a *continuous* function defined on $[-1, 1]$ so that $\int_{-1}^1 f^2(x)dx = 0$. Give a reason why we can or can not conclude that $f(x) = 0$, for every $x \in [-1, 1]$.

(10%) 8. Suppose $f(x)$ is a *differentiable* function defined on $(-\infty, \infty)$ satisfying $f(x+y) = f(x) \cdot f(y)$, for every x and y. Show that $f(x) = a^x$ for some positive number a. (Hint: The function $\ln f(x)$ must have constant derivative.)

(10%) 9. Evaluate the integral $\int\int_D xydxdy$, where

$$D = \{(x, y) \mid 3 \le x + y \le 6,\ 0 \le x - 2y \le 3\}.$$

(10%) 10. Determine the maximum of the function $f(x, y, z) = x - 2y + 3z$ defined on the surface

$$\{(x, y, z) \mid x^2 + y^2 + 5z^2 = 7\}.$$

試題隨卷繳回