

1 (10%)

Find $\lim_{x \rightarrow 0^+} \left(\frac{e^x - 1}{x^2} - \frac{1}{x} \right)$

2 (10%)

Evaluate $\int_0^{\infty} e^{-x} \sin x \, dx$

3 (10%)

Solve the solution $y(x)$ for the following differential equation

$$\frac{dy(x)}{dx} = y(x)(1 - y(x))$$

4 (15%)

Solve the solution $y(x)$ for the following differential equation

$$x^2 \frac{dy(x)}{dx} - 2xy(x) - y(x) = 0$$

with $x \neq 0$ and $y(1) = 1/e$.

5 (10%)

Find the characteristic function for an exponentially distributed random variable with parameter λ .

6 (15%)

Let the joint pdf of two random variables X and Y be

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-x}e^{-y} & \text{for } 0 \leq y \leq x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability of the event: $[X + Y \leq 1]$.

7 (15%)

Given 2D vectors e_1 and e_2 , find a Householder matrix H so that $H(e_1) = e_2$.

8 (15%)

Find an invertible matrix P that can diagonalize the following matrix A by $P^{-1}AP$.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

試題隨卷繳回