

1~8 題為填充題,請依題號,將答案填寫於答案卷上。

1.  $\begin{bmatrix} 2 & 8 \\ 8 & 3 \end{bmatrix} + \begin{bmatrix} 8 & 2 \\ 2 & 7 \end{bmatrix} = \underline{\hspace{2cm}}$  (5%)

2.  $\begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} = \underline{\hspace{2cm}}$  (5%)

3.  $\begin{bmatrix} \cos \frac{\pi}{100} & -\sin \frac{\pi}{100} \\ \sin \frac{\pi}{100} & \cos \frac{\pi}{100} \end{bmatrix}^{100} = \underline{\hspace{2cm}}$  (5%)

4. If  $A \in \mathbf{R}^{n \times n}$  and  $A^2 - A + I_n = 0$ , then  $(A + 2I_n)^{-1} = \underline{\hspace{2cm}}$  (5%)

5. If  $P = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 \\ 9 & 10 & 11 & 12 \\ 16 & 15 & 14 & 13 \end{bmatrix}$ , then  $\text{rank}(P) = \underline{\hspace{2cm}}$ . (5%)

6. If  $A, B \in \mathbf{R}^{3 \times 3}$ ,  $\det(A) = 2$ , and  $\det(B) = -1$ , then

$\det \begin{bmatrix} 2I_3 & B \\ 0 & AB \end{bmatrix} = \underline{\hspace{2cm}}$ . (5%)

7. If  $\mathbf{u} \in \mathbf{R}^n$  and  $\mathbf{u}'\mathbf{u} = [3]$ , then there exist  $k \in \mathbf{R}$  such that

$(I_n + \mathbf{u}\mathbf{u}')^{10} = I_n + k\mathbf{u}\mathbf{u}'$ , where  $k = \underline{\hspace{2cm}}$ . (5%)

8. Let  $B = \begin{bmatrix} 4 & 1 & 0 & 0 & 1 \\ 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 1 & 0 & 0 & 1 & 4 \end{bmatrix}$  and  $\mathbf{x} = \begin{bmatrix} \cos(\theta + \phi) \\ \cos(2\theta + \phi) \\ \cos(3\theta + \phi) \\ \cos(4\theta + \phi) \\ \cos(5\theta + \phi) \end{bmatrix}$

(a)  $\exists a, b, c \in \mathbf{R}$  such that  $B\mathbf{x} - a\mathbf{x} = \begin{bmatrix} b \\ 0 \\ 0 \\ 0 \\ c \end{bmatrix}$ , where  $(a, b, c) = \underline{\hspace{2cm}}$ . (6%)

(Please express  $a, b$ , and  $c$  in terms of  $\theta$  and  $\phi$ .)

(b) All the eigenvalues of  $B$  are  $\underline{\hspace{2cm}}$ . (9%)

見背面

※以下第 9~12 題全對得 10 分，錯一個選項得 5 分，錯二個(含)以上選項得 0 分。

9. 以下敘述何者為正確? 請清楚標示答案，不需說明。
- Suppose that  $A$  and  $B$  are two finite sets. It is possible that the number of different binary relations from  $A$  to  $B$  is not equal to the number of different binary relations from  $B$  to  $A$ .
  - Define a relation  $\mathfrak{R}$  on the set of integers as follows:  $x \mathfrak{R} y$  if and only if  $xy \geq 0$ . Then,  $\mathfrak{R}$  is an equivalence relation.
  - Suppose that  $\mathfrak{R}$  is an equivalence relation on  $A$ . Then the set of all equivalence classes induced by  $\mathfrak{R}$  forms a partition of  $A$ .
  - The Hasse diagram for a total ordering is a chain.
  - It is possible that there are multiple topological orders for a partially ordered set.
10. 以下敘述何者為正確? 請清楚標示答案，不需說明。
- If  $(R, +, \cdot)$  is a ring, then  $(R, +)$  is a commutative group.
  - Every ring  $(R, +, \cdot)$  has the identities for both  $+$  and  $\cdot$ .
  - $(R, +, \div)$  forms a ring, where  $R$  is the set of real numbers,  $+$  is the ordinary addition, and  $\div$  is the ordinary division.
  - The principle of duality holds for both Boolean algebras and rings.
  - If  $(G, \cdot)$  is a finite group and  $H \subseteq G$  is a subgroup of  $G$ , then  $|H|$  can divide  $|G|$ .
11. 以下敘述何者為正確? 請清楚標示答案，不需說明。
- There are 26 integers in  $\{1, 2, \dots, 100\}$  that are not divisible by 2 or 3 or 5.
  - It is known that there are  $n^r$   $r$ -permutations of  $n$  distinct objects with unlimited repetitions. The answer (i.e.,  $n^r$ ) can be expressed as the coefficient of  $x^r$  in  $f(x) = (1 + x + x^2 + x^3 + \dots)^n$ .
  - The general solution to  $a_{n+2} = 4a_{n+1} - 4a_n$ , where  $n \geq 0$  and  $a_0 = 1$ , is  $a_n = 2^n$ .
  - The recurrence equation  $b_n = b_0b_{n-1} + b_1b_{n-2} + \dots + b_{n-2}b_1 + b_{n-1}b_0$ , where  $b_0$  is a given constant, can be solved by the aid of generation functions.
  - Every nonlinear recurrence equation of order three with constant coefficients is solvable.
12. 以下敘述何者為正確? 請清楚標示答案，不需說明。
- (All graphs below are simple and undirected.)
- Two graphs are isomorphic, if they have the same number of vertices, the same number of edges, and the same nonincreasing vertex degree sequence.
  - If a graph has vertex connectivity 3, then it is biconnected.
  - There exists a graph that contains both a Hamiltonian path and an Euler trail.
  - There exists a planar graph of 10 vertices and 14 edges whose planar drawing divides the plane into 8 regions.
  - There exist sufficient-and-necessary conditions for a graph to be planar.
13. Suppose that  $p_1, p_2, \dots, p_6$  denote six people, where every two people are either familiar with or strange to each other. Prove that at least three of  $p_1, p_2, \dots, p_6$  can be found so that they are either familiar with or strange to one another. (10%)