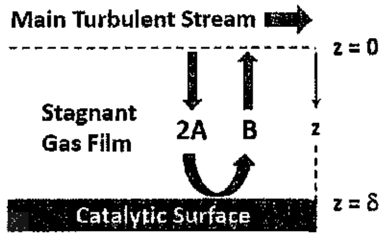
Problem 1 (12%)

Fick's first law of one-dimensional binary diffusion can be expressed as follows.

$$N_{Az} = -cD_{AB}\frac{\partial x_A}{\partial z} + x_A(N_{Az} + N_{Bz})$$

(1) Please give the SI units of (a) molar flux, N_{Az} and (b) diffusivity, D_{AB}. (4%)

(2) Let us now consider a simple model for a catalytic reactor in which a reaction 2A → B is being carried out. Assume that each catalyst particle is surrounded by a stagnant gas film through which A has to diffuse to reach the catalyst surface. At the catalyst surface we assume that the reaction 2A → B occurs instantaneously and that the product B then diffuses back out through the gas film to the main turbulent stream composed of A and B (see right figure).

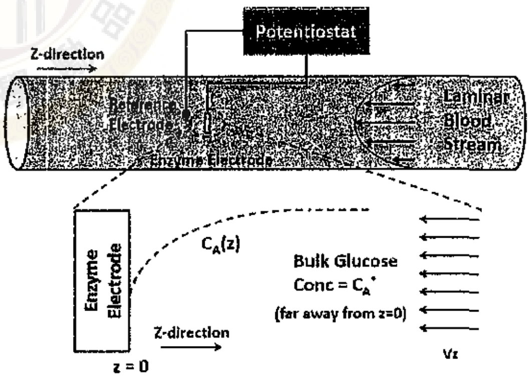


Please show that the molar flux of reactant through the film can be expressed as follows, where δ is the effective gas-film thickness and x_{A0} is the molar fraction of A in the main stream. (8%)

$$N_{Az} = \frac{2cD_{AB}}{\delta}\ln\left(\frac{1}{1 - x_{A0}/2}\right)$$

Problem 2 (23 %)

There is a constant, laminar blood stream flowing through a cylindrical microfluidic channel, and a miniaturized enzyme electrode is placed at the center of the channel to detect the glucose concentration (C_A^*) in the blood stream. The detection is based on the mass-transfer-controlled oxidation of glucose at the enzyme electrode, and the presence of the enzyme electrode does not influence the fully developed velocity profile of the blood stream.



(1) When one-dimensional diffusion and convection are considered, the molar flux of glucose nearby the enzyme electrode can be written as follows.

$$N_A = -D_A\frac{\partial C_A}{\partial z} + C_A v_z$$

Please take advantage of shell mole balance to prove that the equation of change (Fick's 2nd law) nearby the enzyme electrode will be

$$\frac{\partial C_A}{\partial t} = D\frac{\partial^2 C_A}{\partial z^2} - v_z\frac{\partial C_A}{\partial z}$$

Provide all assumptions made for derivation of the equation of change. (5%)

(2) Assume that glucose is completely oxidized (i.e. totally reacted) at the electrode surface (z = 0) but remains a constant bulk concentration C_A^* at the upstream far

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from the electrode. Please solve the glucose concentration profile $C_A(z)$ when the steady state is attained. (7%)

(3) According to Faraday's law, the glucose sensing current (I_A) can be correlated to glucose molar flux (N_A) by the following equation where n, F and A represent number of electron transfer (n = 2 in the present case), charge on one mole of electrons and active electrode area, respectively.

$$I_A = -nFA \times N_A(\text{at } z = 0)$$

Please determine the glucose sensing current as a function of C_A^* *and* v_z. (5%)

(4) The sensitivity (SEN) of a biosensor is defined as the ratio of I_A to C_A^*, i.e., SEN = I_A/C_A^*. If v_z can be regarded as the maximum velocity of the blood stream, what factors will affect the detection sensitivity of this glucose sensor device? (Hint: Recall Hagen-Poiseuille equation for a capillary flow.) (6%)

Problem 3 (15%)

(1) The right figure illustrates a single-stage mixer settler for extraction purpose. F, S, R and E are the flow rates of feed, solvent, raffinate and extract streams, respectively. x_F, y_S, x and y represent the mass fractions of the solute in corresponding streams. Please show that the recovery rate γ(i.e., the fraction of the solute recovered by an extractor) is

$$\gamma = \frac{K(E/R)}{1 + K(E/R)}$$

where K is the distribution coefficient (y = Kx). Provide all assumptions you made. (8%)

(2) For multistage countercurrent extraction, the following equation is convenient for determination of ideal stage number N_p.

$$N_p = \frac{\ln\left[\frac{x_F - y_s/K}{x_{N_p} - y_s/K}(1 - R/KE) + R/KE\right]}{\ln(KE/R)}$$

Consider that penicillin F is to be extracted from the clarified fermentation beer by using pure amyl acetate as solvent at pH 4.0. The distribution coefficient K of the system was found to be 32. The initial concentration of penicillin in the feed is 400 mg/L. The flow rates of the feed (F) and the solvent (S) streams are 640 L/hr and 40 L/hr, respectively. How many ideal stages (countercurrent contact) are required to recover 95 percent of penicillin in the feed? (Hint: log 2 = 0.301; log 3 = 0.477; log (A + B) ≅ log (A) if B << A) (7%)

Problem 4 (20%)

(1) Derive the differential equations of motion for a fluid of constant viscosity and density, which is flowing over an impulsively accelerated, infinitely long *horizontal flat plate. Assume that the flow is laminar. Please define all the* variables and state your assumptions. (5%)

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(2) Given a two-dimensional flow described by $u = x^2 + 2x - 4y$, $v = -2xy - 2y$. Does this satisfy continuity? Find the expression for the stream function. (10%)
(3) Determine the terminal velocity of a small cubic of size d falling in the solution of constant density ρ and viscosity μ. Define all your variables and assumptions. (5%)

Problem 5 (12%)

(1) Consider blood flows in small capillaries of radius R and of finite length $2L$ with permeable wall to promote fluid exchange across the wall. Assume the steady laminar axisymmetric flow of an incompressible Newtonian fluid through the capillaries. Determine equations of momentum and continuity for the system. Please define all your variables and state your assumptions. (6%)
(2) Perform the dimensional analysis for the governing equations. Please express the equations in terms of the following dimensionless variables and parameters. (6%)

$$r^* = \frac{r}{R},\ z^* = \frac{z}{L},\ v_z^* = \frac{v_z}{R^2\alpha/\mu L},\ v_r^* = \frac{v_r}{R\alpha/\mu},\ p^* = \frac{p - p_0}{\alpha},\ \beta = \frac{R}{L},\ G = \frac{R^2\rho\alpha}{\mu^2}.$$

(Hint: the general equation of continuity and Navier-Stokes equation for cylindrical coordinates (r,θ,z) are as follows:

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z\ )$$

Problem 6 (18%)

(1) Give the formulas for Biot and Prandtl numbers, and explain their physical meanings. (8%)
(2) A cold room of 5x3x4 m (Length x height x width) is used to refrigerate food. The temperature inside the room is kept at $1^\circ C$ and the outside temperature is $25^\circ C$. The ceiling and walls are composed of three layers of materials from inside to the outside: 2 cm of plastic (k=14 W/(mK)), 12 cm of asbestos (k=0.5 W/(m$^\circ C$)) and 20 cm of wood (k=1.15 W/(m$^\circ C$)). The convective heat transfer coefficient inside and outside the chamber is 10 W/(m$^\circ C$). The floor is made of 10 cm concrete (k=1.1 W/(m$^\circ C$)). The temperature of the ground is $10^\circ C$. What is the amount of heat coming from the outside to the cold room in 1 hour? (10%)

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