

Note: You can use either Chinese or English to answer the questions below. The total number of points is 100. The points for each problem is indicated in the parenthesis. You need to show all the work to receive the points.

1. Let X be a random variable with pdf $f_X(x) = 6x(1-x)$, $0 < x < 1$.
 - (a) (17 points) Determine the moment generating function of X .
 - (b) (17 points) Define $Y = \ln\left(\frac{X}{1-X}\right)$, where \ln is the nature logarithm. Find the pdf of Y .
2. (16 points) Let A , B , and C be events; A and C are independent. Show that
$$P(A \cup B|C) = P(A) + P(B|C) - P(A)P(B|A \cap C).$$
3. Let X_1, \dots, X_n be a random sample from a normal distribution with known mean, μ , and unknown variance $\sigma^2 > 0$.
 - (a) (15 points) Find the maximum likelihood estimator (MLE) of σ^2 .
 - (b) (20 points) Prove or disprove each of the following:
 - i. The MLE is unbiased.
 - ii. The MLE is consistent.
4. (15 points) Let X_1, \dots, X_{10} be a random sample from a $N(\mu, \sigma^2)$ distribution. Find (in simplified form) the form of the most powerful level α test of

$$H_0: \mu = 0, \sigma^2 = 1 \quad \text{against} \quad H_1: \mu = 1, \sigma^2 = 4$$

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