1. (14%) Find the general solution.

   (a) $\cos(x+y)dx+[3y^2+2y+\cos(x+y)]dy=0$

   (b) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+4y=1$

2. (10%) Suppose that in Winter the daytime temperature in a certain office building is maintained at 70°F. The heating is shut off at 10 p.m. and turned on again at 6 a.m.. On a certain day the temperature inside the building at 2 a.m. was found to be 65°F and the outside temperature was about 45°F. What was the temperature inside the building when the heat was turned on at 6 a.m.?

   (Hint: by Newton's law: $\frac{dT}{dt}=k(T-T_A)$)

3. (10%) Find the power series solution near $x=0$

   $(x^2+4)\frac{d^2y}{dx^2}+xy=x+2$

4. (6%) Find the inverse Laplace Transform

   $\frac{s-6}{(s-1)^2+4}$

5. (10%) Show that

   (a) $\mathcal{L}\{\frac{df(t)}{dt}\}=s\mathcal{L}\{f(t)\}-f(0)$

   If f is continuous for all $t\geq 0$ and satisfy the growth restriction and $\frac{df}{dt}$ is piecewise continuous on every finite interval on the semi-axis $t\geq 0$

   (b) $\mathcal{L}\{\frac{d\delta(t)}{dt}\}=s$

   where $\delta(t)$ = delta function

6. (14 %) If a vector field $\underline{F}=2xy\underline{i}+(x^2-1/y)\underline{j}$ $(y\neq 0)$, C: the path connecting (1,3) and (2,2).

   (a) Please show that $\underline{F}$ is a conservative vector field. Since $\underline{F}$ is a conservative vector field, there exists a potential function ϕ. Please also show the relationship between $\underline{F}$ and ϕ.

   (b) Please calculate ϕ and $\int_C \underline{F}\bullet d\underline{r}$.

見背面

7. (21 %) Consider $f(x)=\frac{x^2}{2}$ for $-\pi \le x \le \pi$,

(a) Find the Fourier series of *f(x)*

(b) Use the answer of (a) to evaluate the value of $\sum_{n=1}^{\infty} 1/n^2$.

(c) Use the answer of (a) to evaluate the value of $\sum_{n=1}^{\infty} (-1)^n/n^2$.

8. (15%) Solve the following problem:

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} \qquad \text{for } 0 \le x \le 1$$

$$u(0,t) = u(1,t) = 0 \qquad \text{for } t \ge 0$$

$$u(x,0) = \frac{1}{2}\sin \pi x + \frac{1}{4}\sin 3\pi x \qquad \text{for } 0 \le x \le 1$$

$$\frac{\partial u}{\partial t}(x,0) = 0 \qquad \text{for } 0 \le x \le 1$$

試題隨卷繳回